18 March 2007 Hideyuki Tatsuno

Compton tail fitting

I. Simulation

II. Fitting

I. Simulation

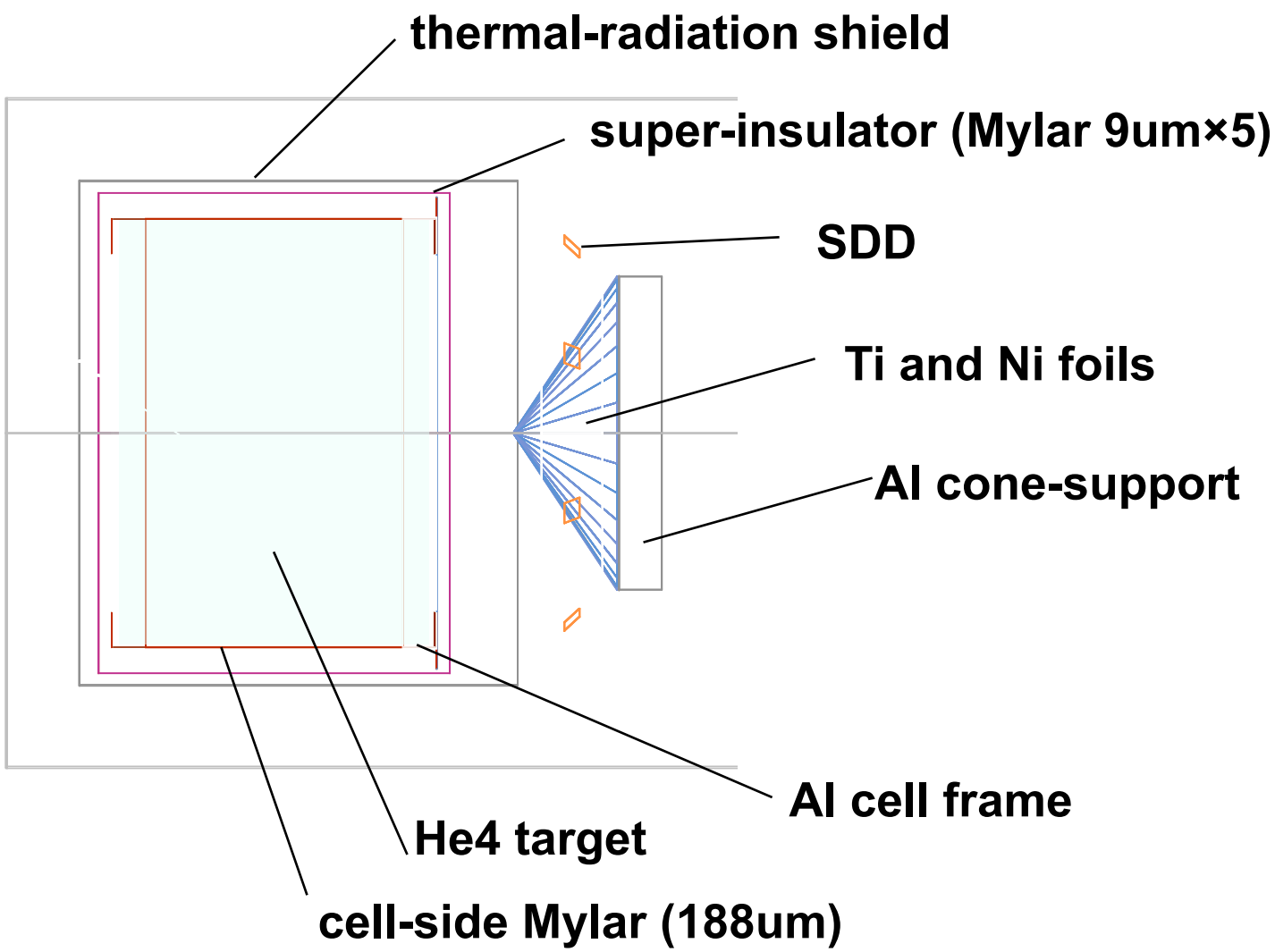
Geant 4 tool-kit with the LECS package

I got a response from the EGS4 developers (Prof. Hirayama and Prof. Namito) .
They said the simulation error of the cross section of bound-Compton scattering
and Rayleigh scattering in He4 is only several %. We don't have to worry about them.

1. Geometry

2. Stopped Kaon distribution

1. Geometry (simulation)



not considered...

Cu pipe

LN2 pipe

Cell frame
(top and bottom)

Inner pure-Al foils

7.5 um-thick-Kapton
tape (front of SDD)

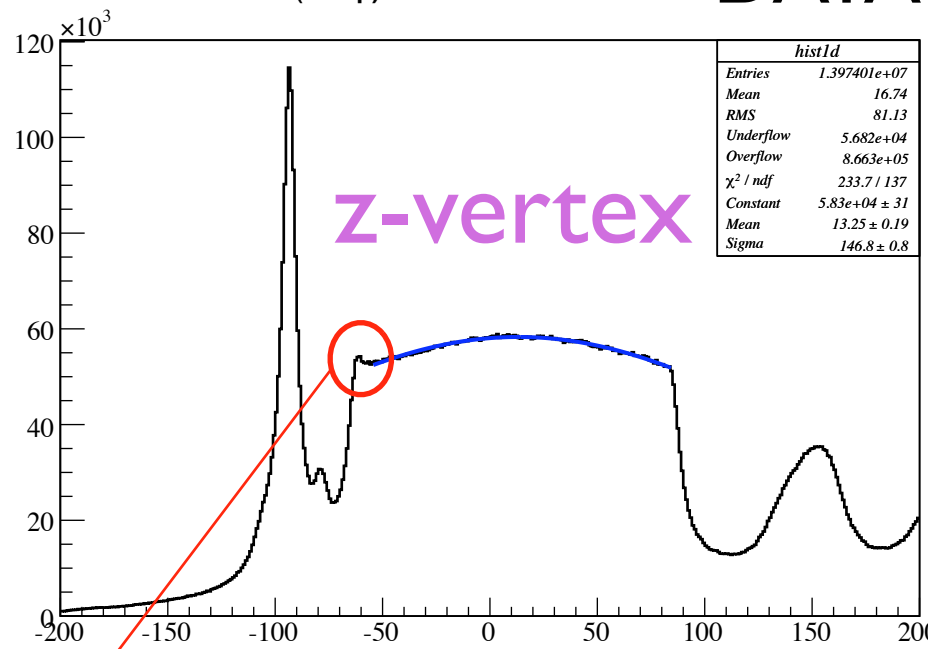
2. Stopped Kaon distribution (1st cycle)

VDC + PDC tracking

"Kstop" definition is same as the okada-san's report on 6 Oct, 2006

x,y and z distributions were fitted by Gaussians respectively

DATA



This spike was excluded

x-vertex

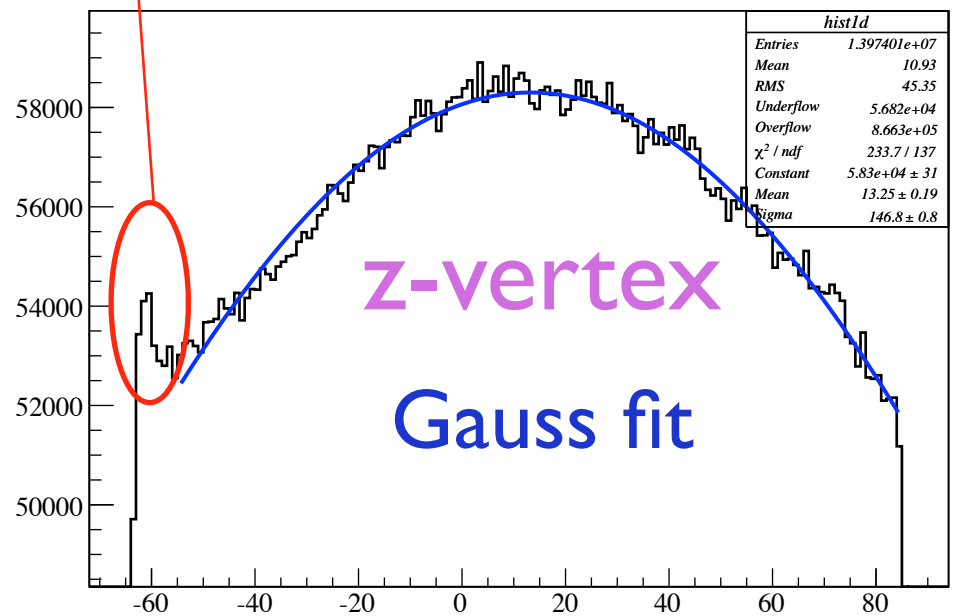
NO.	NAME	VALUE	ERROR
1	Constant	1.60365e+03	4.67581e+00
2	Mean	9.65175e-02	1.41729e-01
3	Sigma	5.39760e+01	1.42565e-01

y-vertex

NO.	NAME	VALUE	ERROR
1	Constant	1.74664e+03	5.01736e+00
2	Mean	1.05829e-01	1.20197e-01
3	Sigma	4.83348e+01	1.10632e-01

z-vertex

NO.	NAME	VALUE	ERROR
1	Constant	5.83017e+04	3.08709e+01
2	Mean	1.32485e+01	1.93959e-01
3	Sigma	1.46798e+02	7.82423e-01



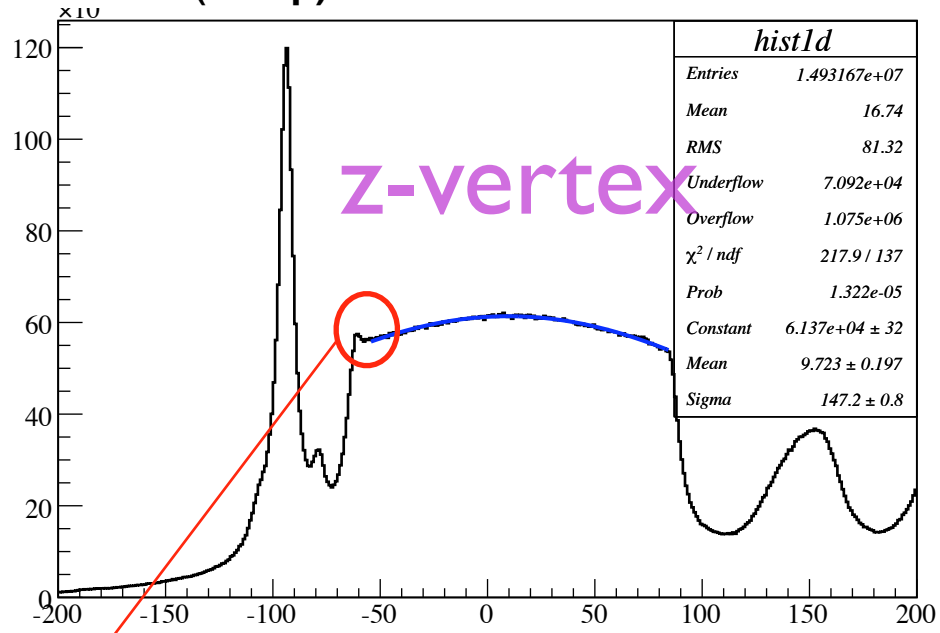
2. Stopped Kaon distribution (2nd cycle)

DATA

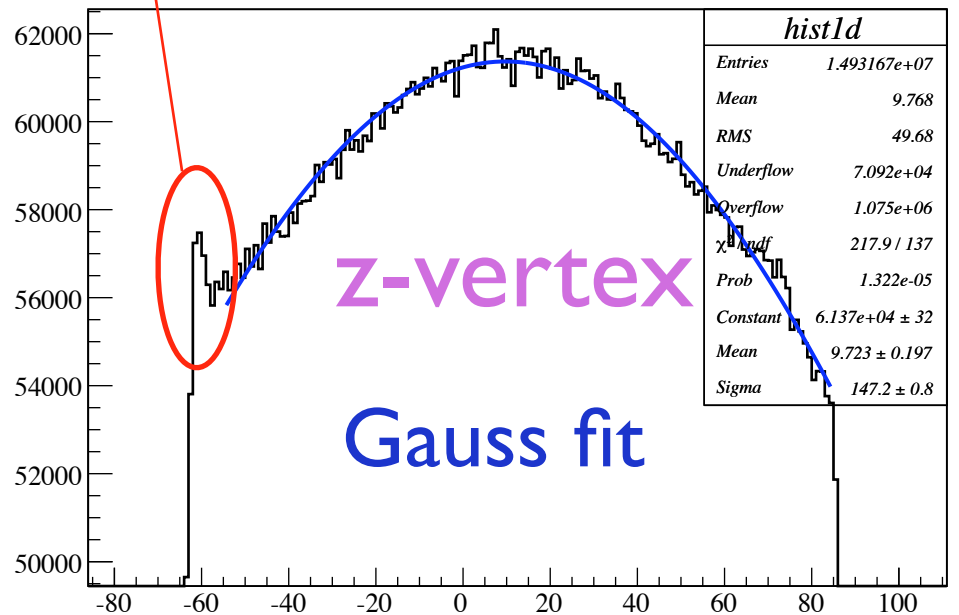
VDC + PDC tracking

"Kstop" definition is same as the okada-san's report on 6 Oct, 2006

x,y and z distributions were fitted by Gaussians respectively



This spike was excluded



x-vertex

NO.	NAME	VALUE	ERROR
1	Constant	9.71193e+02	3.64484e+00
2	Mean	-5.30472e-02	1.84166e-01
3	Sigma	5.44995e+01	1.88526e-01

y-vertex

NO.	NAME	VALUE	ERROR
1	Constant	1.07039e+03	3.91987e+00
2	Mean	-1.02382e-01	1.52559e-01
3	Sigma	4.79061e+01	1.37566e-01

z-vertex

NO.	NAME	VALUE	ERROR
1	Constant	6.13680e+04	3.15142e+01
2	Mean	9.72280e+00	1.96770e-01
3	Sigma	1.47217e+02	7.68956e-01

II. Fitting

Compton tail fit

1. Fit function

The tail distribution is the convolution of an exponential function with a Gaussian and given by

$$T(i, E_{jk}) = \frac{Gain}{2\beta\sigma_{jk}} e^{\frac{E_i - E_{jk}}{\beta\sigma_{jk}} + \frac{1}{2\beta^2}} \operatorname{erfc}\left(\frac{E_i - E_{jk}}{\sqrt{2}\sigma_{jk}} + \frac{1}{\sqrt{2}\beta}\right) \qquad (7)$$

X-Ray Spectrom. 2003; 32: 434 – 441

2. Parameters

```
FCN=72.3163 FROM MINOS     STATUS=SUCCESSFUL    144 CALLS         231 TOTAL
                     EDM=8.5463e-16    STRATEGY= 1      ERROR MATRIX ACCURATE
 EXT PARAMETER                                PARABOLIC          MINOS ERRORS
 NO.   NAME      VALUE                          ERROR       NEGATIVE       POSITIVE
  1  G mean [eV]   6.41354e+03   1.93418e+00  -1.95222e+00   1.91645e+00
  2  G sigma [eV]   8.79822e+01   1.14824e+00  -1.14720e+00   1.14867e+00
  3  Area          1.32900e+05   1.15322e+03  -1.15322e+03   1.15322e+03
  4  Slope         8.75844e-01   3.11523e-02  -3.11170e-02   3.11887e-02
```

Area should be divided by bin-width

3. Histogram

Simulation (δ-function)

⇩

Gauss smearing

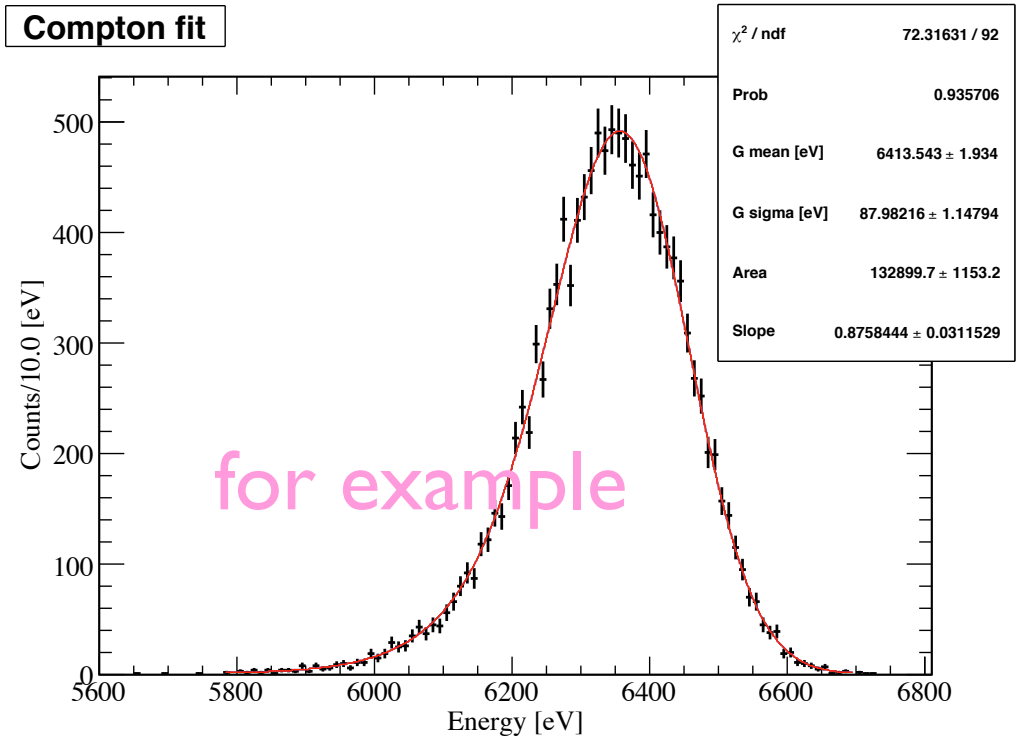
σ was calculated from data using constant noise and Fano factor

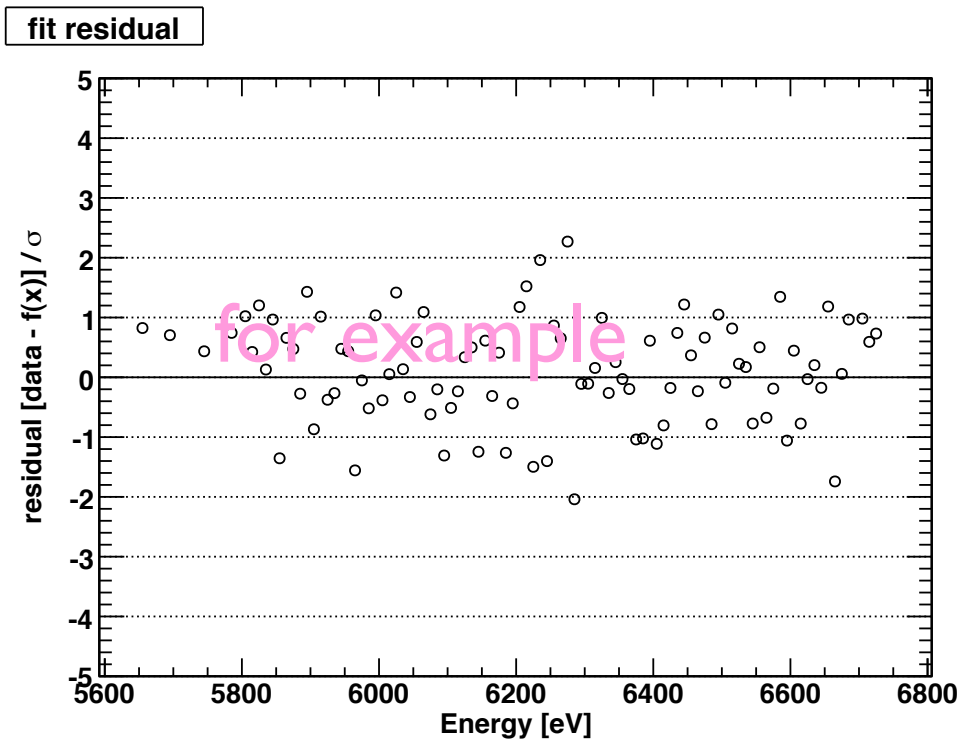
$$\sigma = \sqrt{\mathrm{Noise}^2 + \mathrm{Fano} \cdot w \cdot E}$$

	Noise (eV)	Fano
1st cycle	56.2±0.7	0.145±0.004
2nd cycle	54.5±0.7	0.128±0.004

after iterative fit

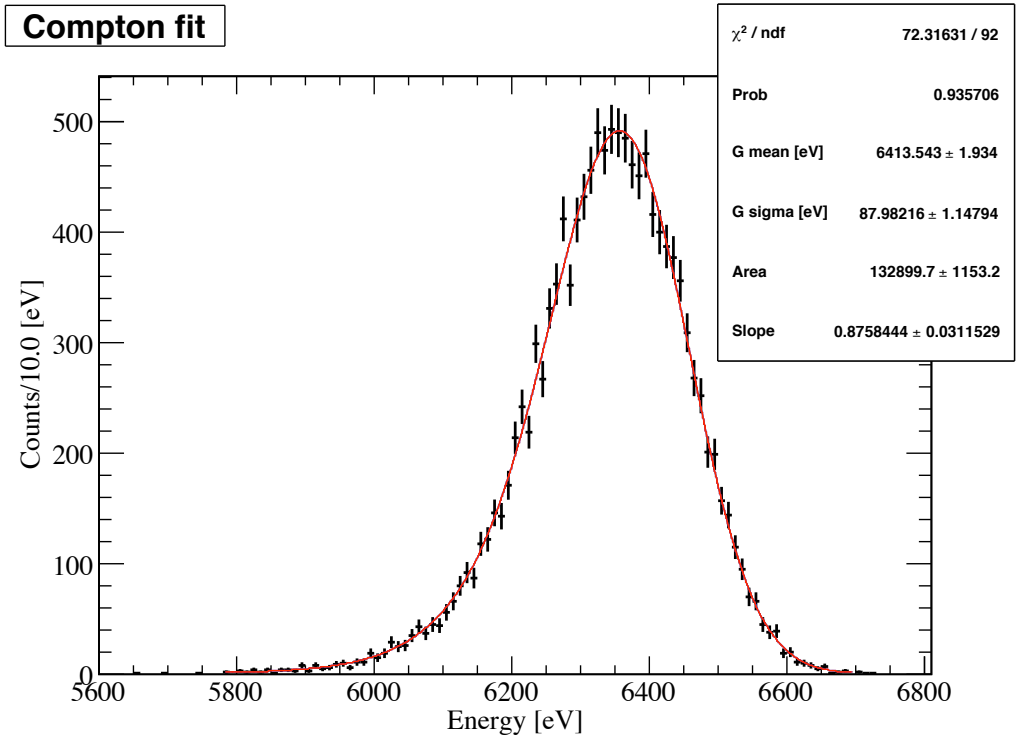
a little better than Tatsuno's thesis

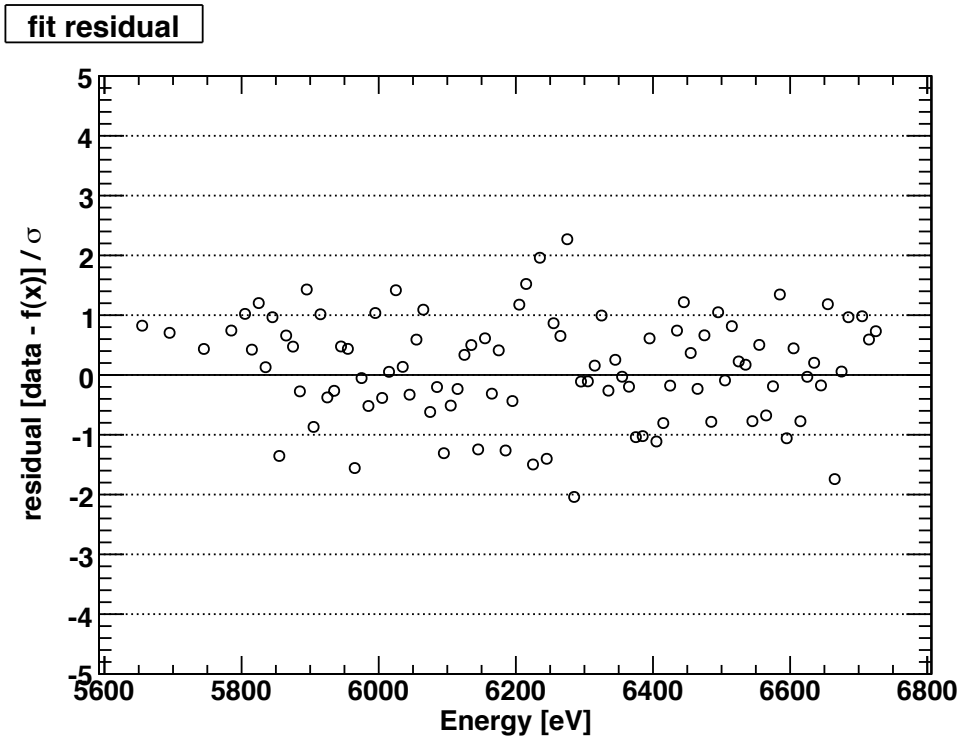




KHeX L alpha (6464.0 eV) fit

1st cycle





1st cycle KHeX La (6464 eV)

only Compton tail

Bin-width : 10 eV
Fit region : 5600-6800 eV

Chisqr/NDF = 72.3/92

Good fit !

1st cycle KHeX La (6464 eV)

Simulation

[KHeX L-alpha (6464eV) 1st cycle]
Number of Events
Total = 110346 +- 332.184
Normal = 86553
Compton = 13385 +- 115.694
Rayleigh = 10155
Compton other + escaped Rayleigh = 253

Ratios
Compton/Total = 0.1213 +- 0.00111023
Compton/(Normal+Rayleigh) = 0.138406 +- 0.00127643

only Compton tail

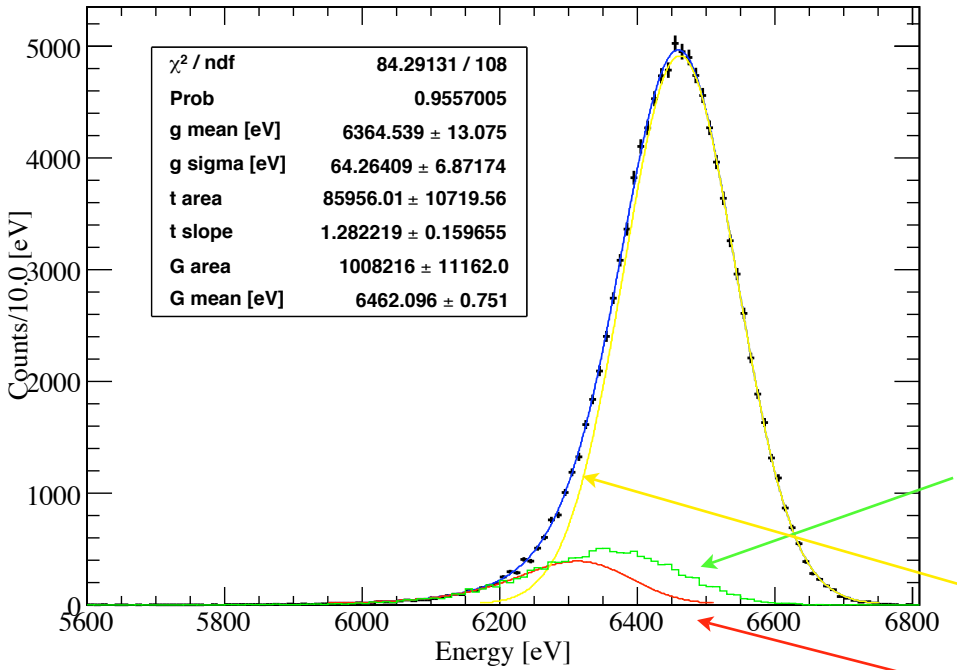
number of Compton event

÷10 : area of the fit function

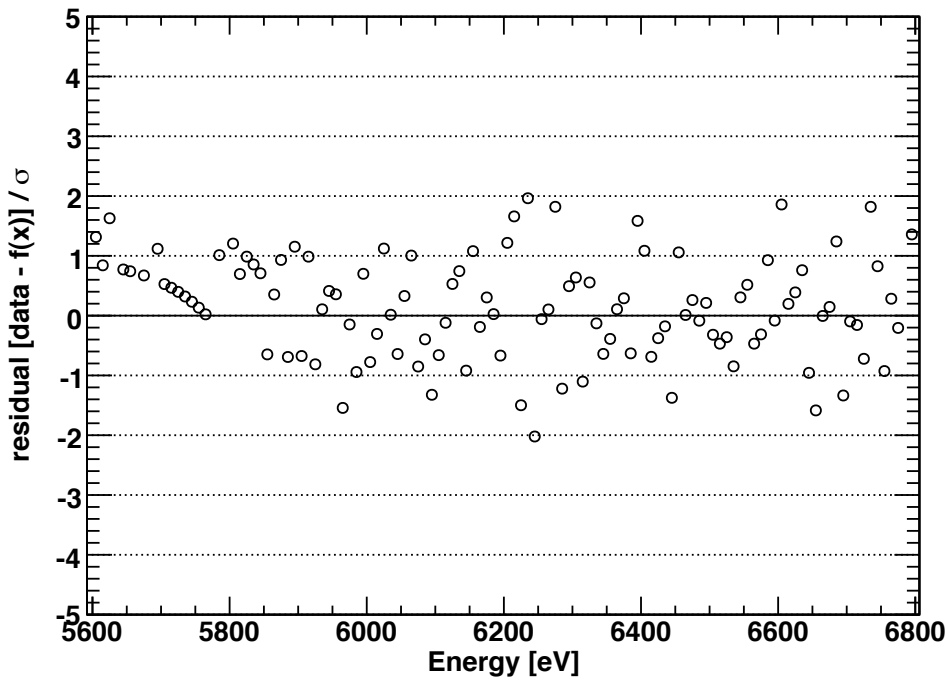
→ good agreement

```
FCN=72.3163 FROM MINOS     STATUS=SUCCESSFUL    144 CALLS         231 TOTAL
                     EDM=8.5463e-16    STRATEGY= 1      ERROR MATRIX ACCURATE
 EXT PARAMETER                   PARABOLIC         MINOS ERRORS
 NO.   NAME      VALUE            ERROR       NEGATIVE       POSITIVE
  1  G mean [eV]   6.41354e+03   1.93418e+00  -1.95222e+00   1.91645e+00
  2  G sigma [eV]   8.79822e+01   1.14824e+00  -1.14720e+00   1.14867e+00
  3  Area          1.32900e+05   1.15322e+03  -1.15322e+03   1.15322e+03
  4  Slope         8.75844e-01   3.11523e-02  -3.11170e-02   3.11887e-02
```

Total fit



fit residual



1st cycle KHeX La (6464 eV)

Γ=0 eV : pure Gaussian

Total fit with free Compton tail

simulated Compton

fitted Main Gaussian

fitted Compton : smaller than the simulated histogram

Bin-width : 10 eV

Fit region : 5600-6800 eV

Chisqr/NDF = 84.3/108

main Gauss mean = 6462.1±0.7

~2eV lower than the generated energy

1st cycle KHeX La (6464 eV)

Γ=0 eV : pure Gaussian

Simulation

[KHeX L-alpha (6464eV) 1st cycle]
Number of Events
Total = 110346 +- 332.184
Normal = 86553
Compton = 13385 +- 115.694
Rayleigh = 10155
Compton other + escaped Rayleigh = 253

Ratios
Compton/Total = 0.1213 +- 0.00111023
Compton/(Normal+Rayleigh) = 0.138406 +- 0.00127643

Total fit with free Compton tail

Compton area = 8595.6
(Normal+Ray) area = 100822

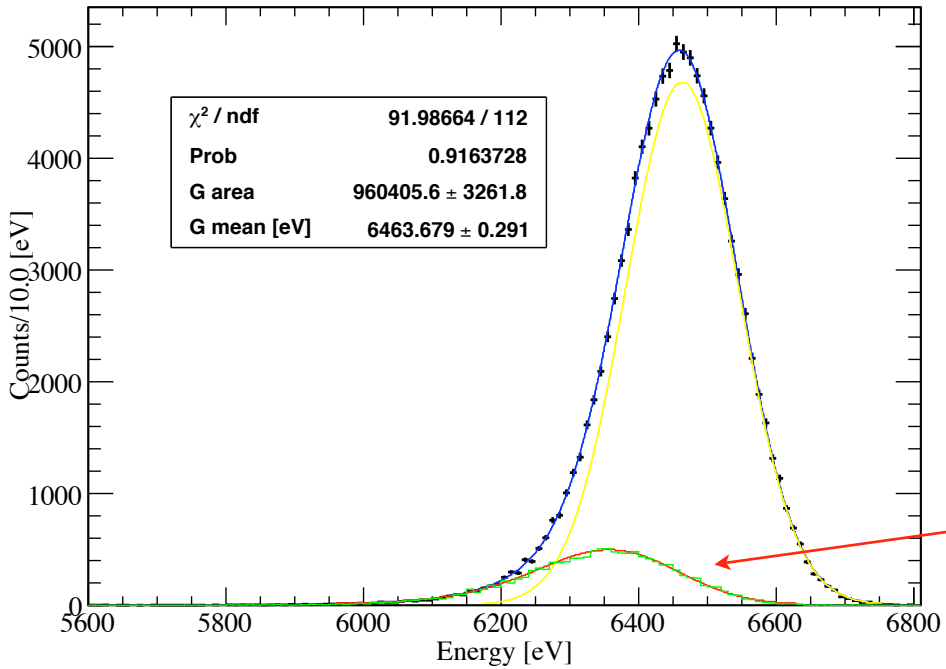
Ratio
Compton/(Normal+Ray) = 0.0852556

too small !

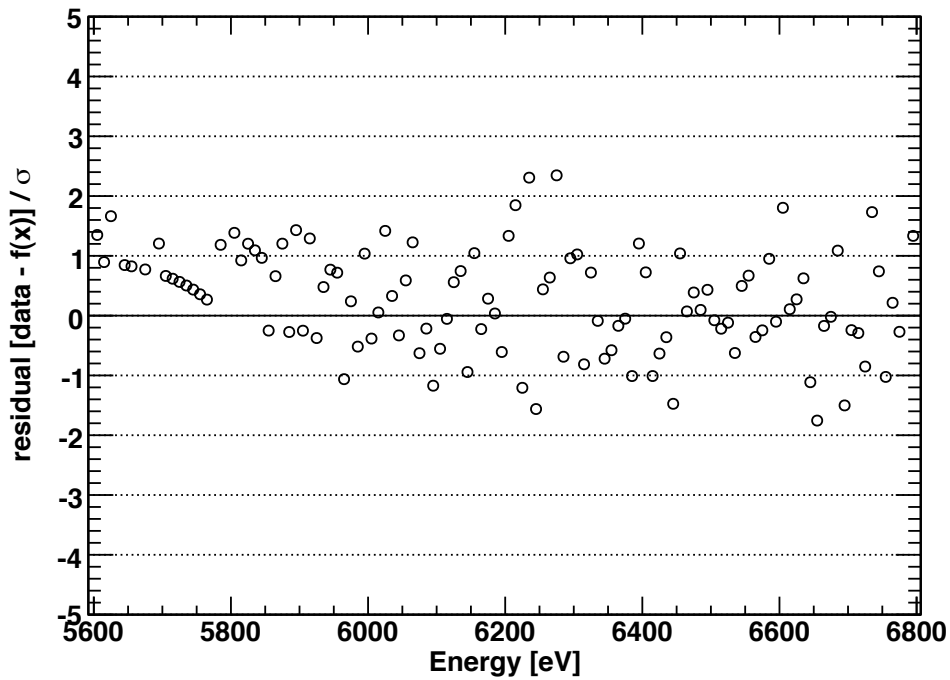
FCN=84.2913 FROM MINOS STATUS=SUCCESSFUL 1524 CALLS 1656 TOTAL
EDM=1.1759e-08 STRATEGY= 1 ERROR MATRIX ACCURATE

EXT NO.	PARAMETER NAME	VALUE	PARABOLIC ERROR	MINOS ERRORS NEGATIVE	MINOS ERRORS POSITIVE
1	g mean [eV]	6.36454e+03	1.24264e+01	-1.08203e+01	1.53287e+01
2	g sigma [eV]	6.42641e+01	6.70386e+00	-6.18081e+00	7.56267e+00
3	t area	8.59560e+04	9.96500e+03	-8.29126e+03	1.31479e+04
4	t slope	1.28222e+00	1.57929e-01	-1.56791e-01	1.62519e-01
5	G area	1.00822e+06	1.04184e+04	-1.34068e+04	8.91719e+03
6	G mean [eV]	6.46210e+03	7.29986e-01	-6.69873e-01	8.32038e-01
7	G sigma [eV]	8.19596e+01	fixed		

Total fit



fit residual



1st cycle KHeX La (6464 eV)

Γ=0 eV : pure Gaussian

Total fit with <u>fixed Compton tail</u>

Tail parameters were fixed

Bin-width : 10 eV
Fit region : 5600-6800 eV

Chisqr/NDF = 92.0/112

main Gauss mean = 6463.7±0.3

Good !

1st cycle KHeX La (6464 eV)

Γ=0 eV : pure Gaussian

Simulation

[KHeX L-alpha (6464eV) 1st cycle]
Number of Events
Total = 110346 +- 332.184
Normal = 86553
Compton = 13385 +- 115.694
Rayleigh = 10155
Compton other + escaped Rayleigh = 253

Ratios
Compton/Total = 0.1213 +- 0.00111023
Compton/(Normal+Rayleigh) = 0.138406 +- 0.00127643

Total fit with fixed Compton tail

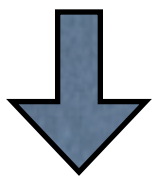
Compton area = 13290
(Normal+Ray) area = 96040.6

Ratio
Compton/(Normal+ray) = 0.138379

FCN=91.9866 FROM MINOS STATUS=SUCCESSFUL 24 CALLS 63 TOTAL
EDM=3.52502e-08 STRATEGY= 1 ERROR MATRIX ACCURATE

EXT NO.	PARAMETER NAME	VALUE	PARABOLIC ERROR	MINOS ERRORS NEGATIVE	POSITIVE
1	g mean [eV]	6.41354e+03	fixed		
2	g sigma [eV]	8.79800e+01	fixed		
3	t area	1.32900e+05	fixed		
4	t slope	8.75800e-01	fixed		
5	G area	9.60406e+05	3.26179e+03	-3.26212e+03	3.26144e+03
6	G mean [eV]	6.46368e+03	2.91185e-01	-2.91213e-01	2.91158e-01
7	G sigma [eV]	8.19596e+01	fixed		

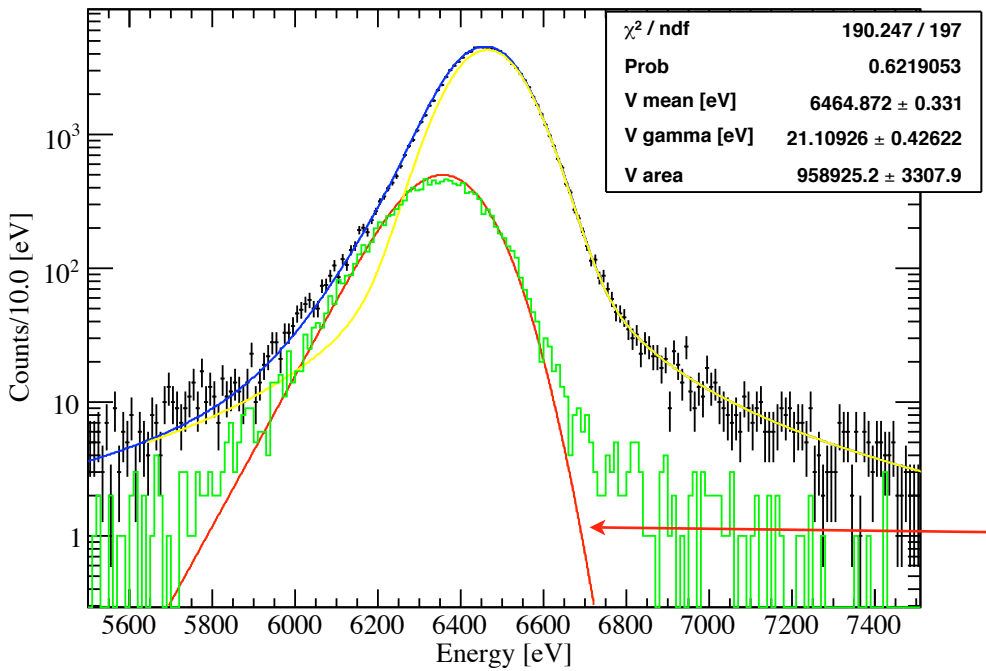
The generated X-ray energy was reproduced by the fitting with fixed Compon tail (Γ=0).



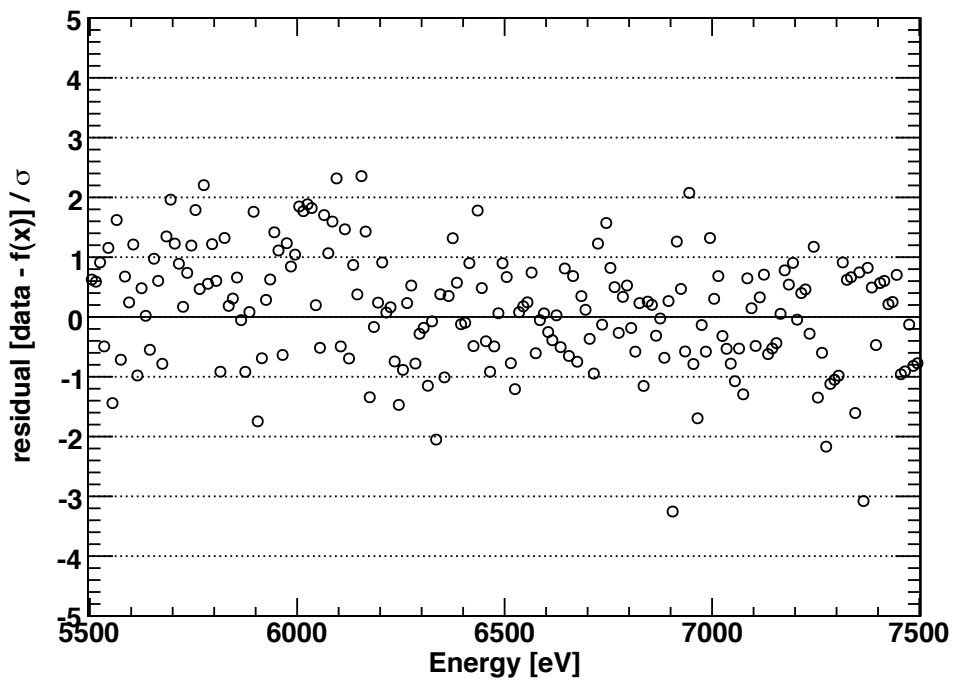
How about Voigt smearing ?

For example Γ=20 eV

Total fit (Voigt)



fit residual



1st cycle KHeX La (6464 eV)

Γ=20 eV : Voigt profile

Total fit with <u>fixed Compton tail</u>

Tail parameters were fixed from the **Gauss*Exp convolution (Not Voigt)**

Bin-width : 10 eV
Fit region : 5500-7500 eV

mean = 6464.9±0.4
gamma = 21.1±0.3

systematics error ~ 1 eV

1st cycle KHeX La (6464 eV)

Γ=20 eV : Voigt profile

Total fit with fixed Compton tail

Simulation

[KHeX L-alpha (6464eV) 1st cycle]
Number of Events
Total = 110346 +- 332.184
Normal = 86553
Compton = 13385 +- 115.694
Rayleigh = 10155
Compton other + escaped Rayleigh = 253

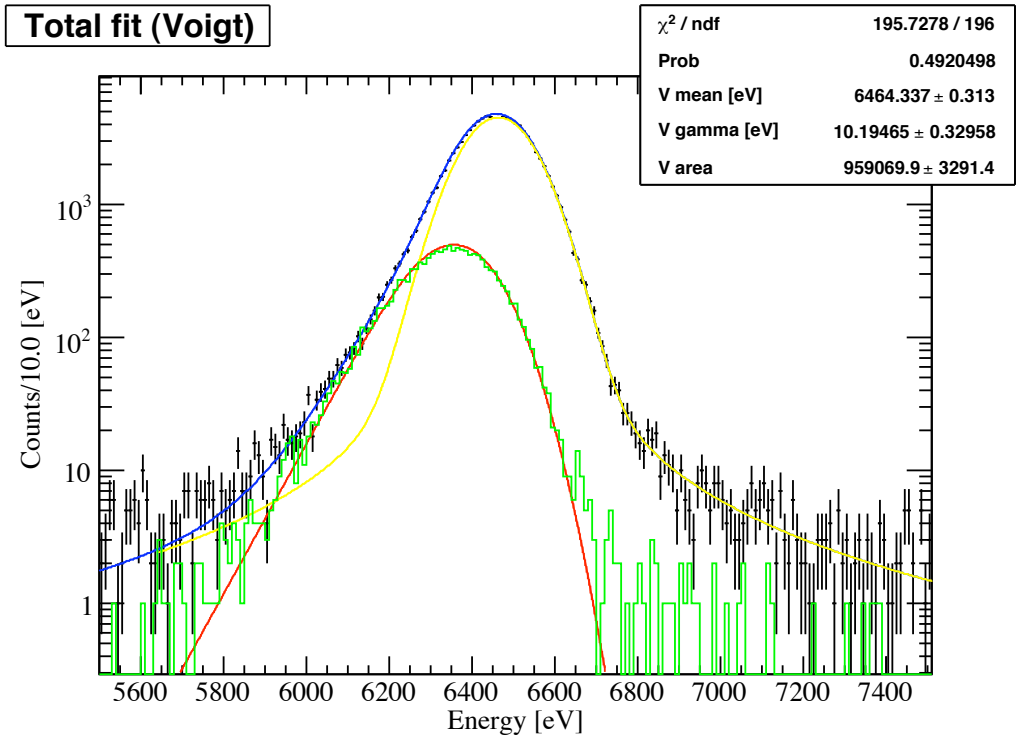
Ratios
Compton/Total = 0.1213 +- 0.00111023
Compton/(Normal+Rayleigh) = 0.138406 +- 0.00127643

```
Compton  area      = 13290
(Normal+Ray) area = 95892.5

Ratio
Compton/(Normal+Ray) = 0.138593
```

```
 FCN=190.247 FROM MINOS     STATUS=SUCCESSFUL     60 CALLS         118 TOTAL
                     EDM=1.09751e-08    STRATEGY= 1      ERROR MATRIX
ACCURATE
  EXT PARAMETER                   PARABOLIC         MINOS ERRORS
  NO.   NAME      VALUE            ERROR      NEGATIVE      POSITIVE
   1  g mean [eV]   6.41354e+03     fixed
   2  g sigma [eV]   8.79800e+01     fixed
   3  t area       1.32900e+05     fixed
   4  t slope      8.75800e-01     fixed
   5  V mean [eV]   6.46487e+03   3.30832e-01  -3.30906e-01   3.30933e-01
   6  V sigma [eV]   8.19596e+01     fixed
   7  V gamma [eV]   2.11093e+01   4.26216e-01  -4.26072e-01   4.26368e-01
   8  V r          4.00000e+00     fixed
   9  V area       9.58925e+05   3.30784e+03  -3.30803e+03   3.30767e+03
```

Γ dependence

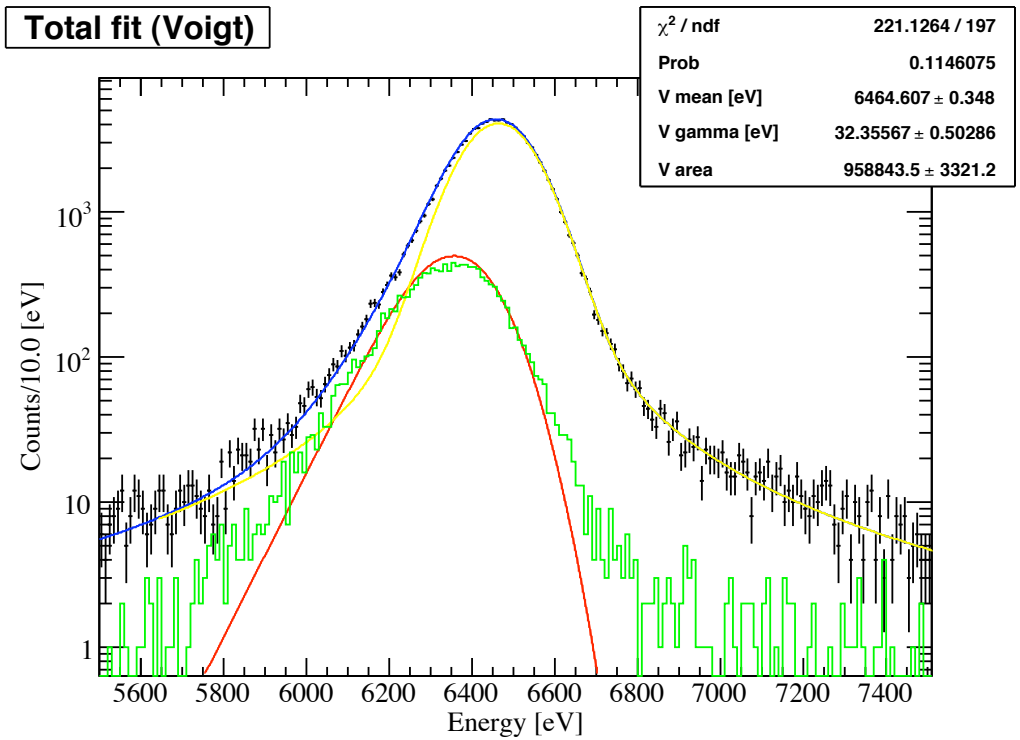


1st cycle KHeX La (6464 eV)

Γ=10 eV : Voigt profile

mean = 6464.3±0.3

gamma = 10.2±0.3

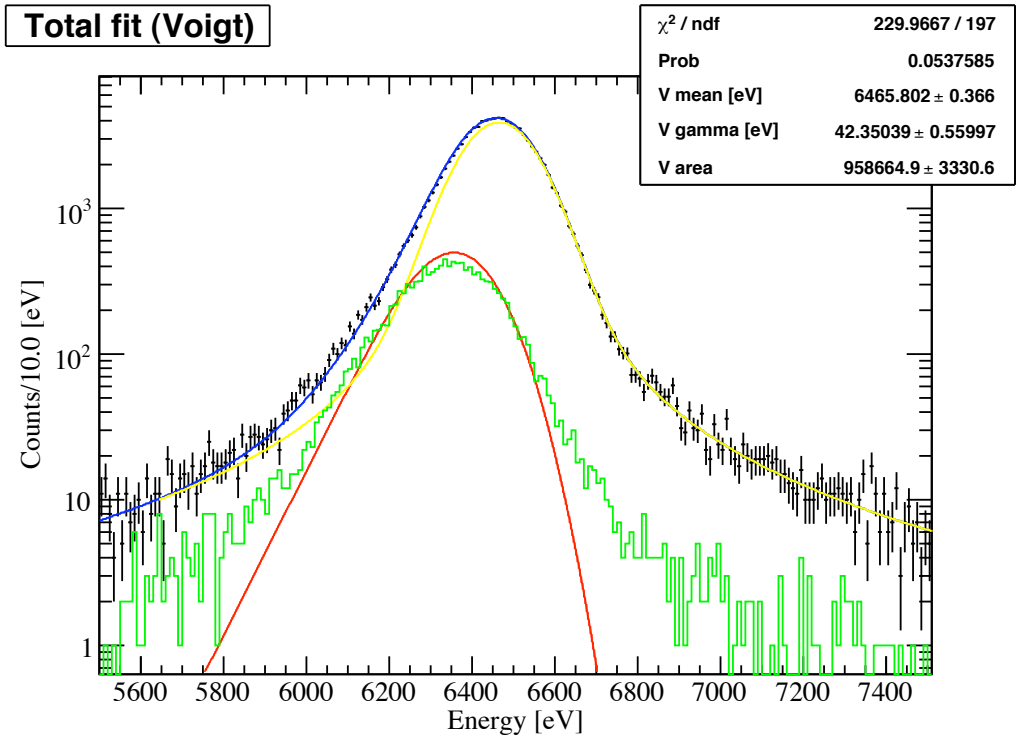


1st cycle KHeX La (6464 eV)

Γ=30 eV : Voigt profile

mean = 6464.6±0.3

gamma = 32.4±0.5

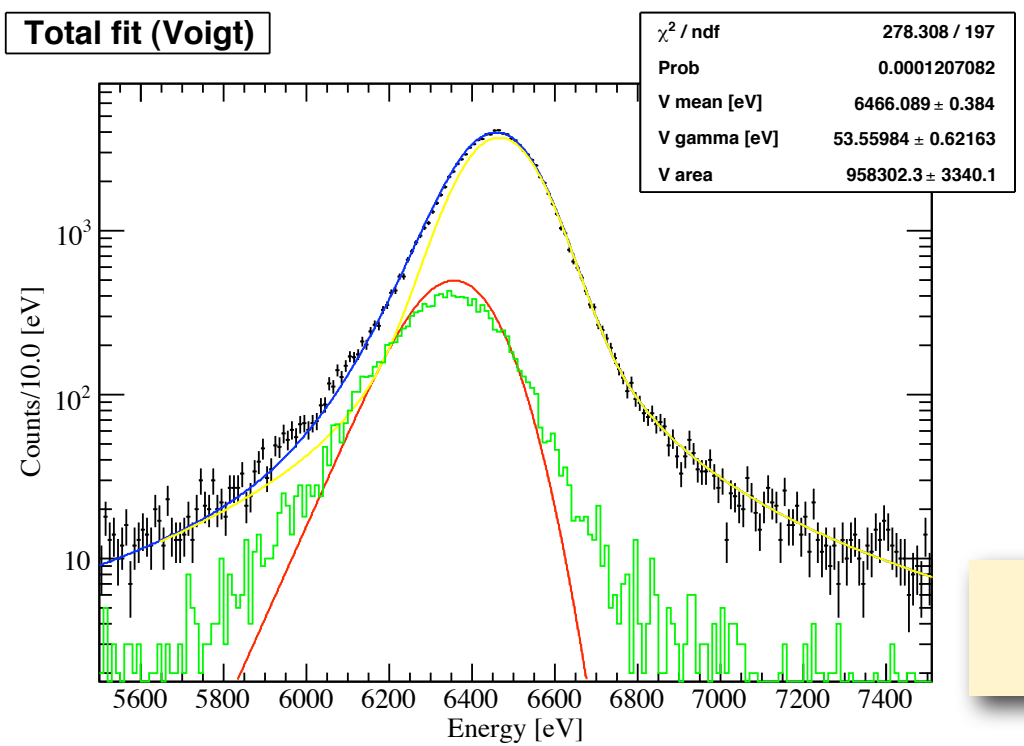


1st cycle KHeX La (6464 eV)

Γ=40 eV : Voigt profile

mean = 6465.8±0.4

gamma = 42.4±0.6



1st cycle KHeX La (6464 eV)

Γ=50 eV : Voigt profile

mean = 6466.1±0.4

gamma = 53.6±0.6

The shift has a systematic error (~2 eV) in large Γ.

Summary

Compton tail can be fitted well by a Exp*Gauss convolution function.

Fixed-Compton-tail fit is reasonable to reproduce the generated photon energy.

Voigt smearing with fixed-Compton has ~±1 eV and ~±1 eV systematics error for the shift and width, respectively (at Γ=20 eV, depend on the Γ).

→ **Since Γ should be determined experimentally, we have to consider the** conservative **systematics for the final results.**

Compton tail correction will be done soon!