Systematic Error of Width ?

Two types

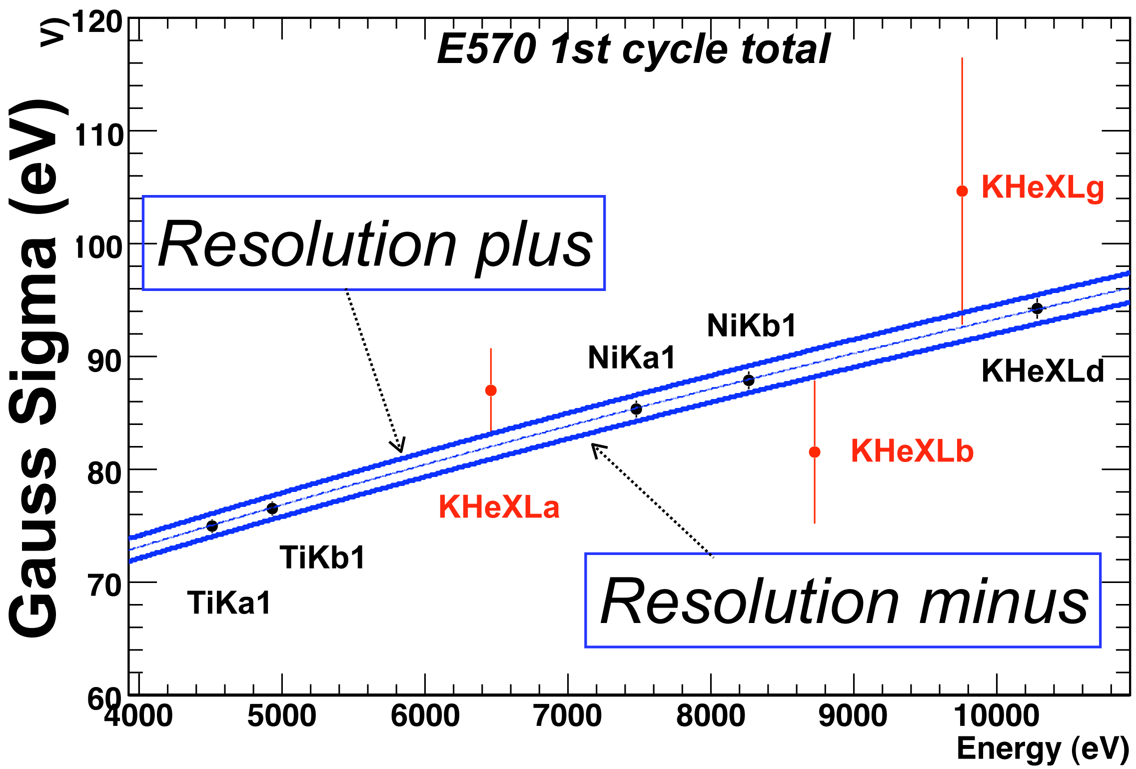
"Resolution +"

$$\sigma_+ = \sqrt{N_+^2 + F_+ wE}$$

"Resolution –"

$$\sigma_- = \sqrt{N_-^2 + F_- wE}$$

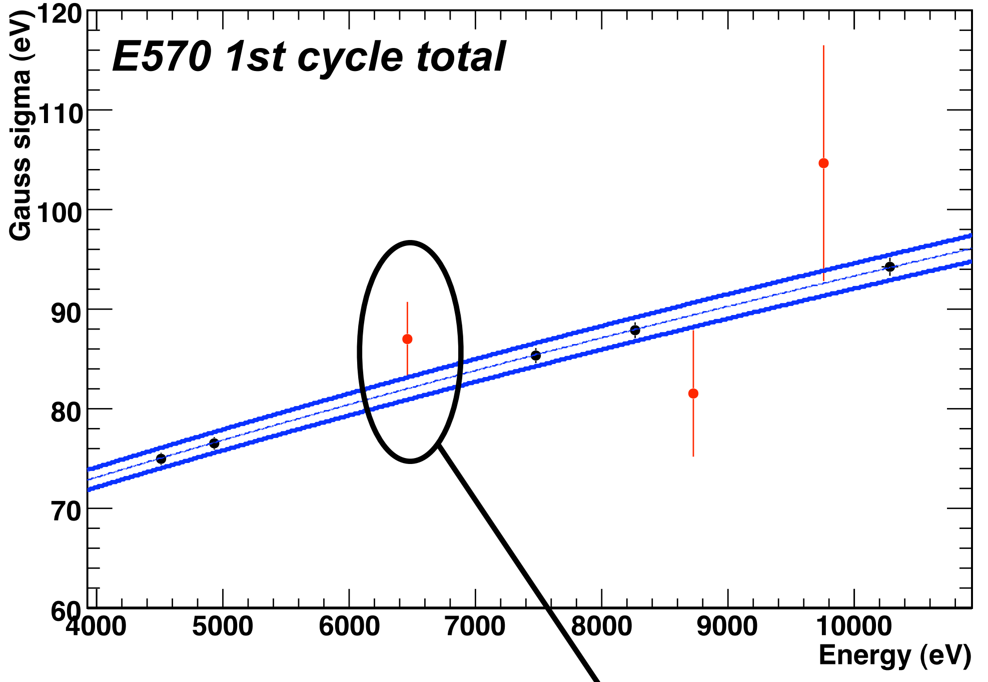
$$N_+ = N + \sigma_N$$
$$F_+ = F + \sigma_F$$

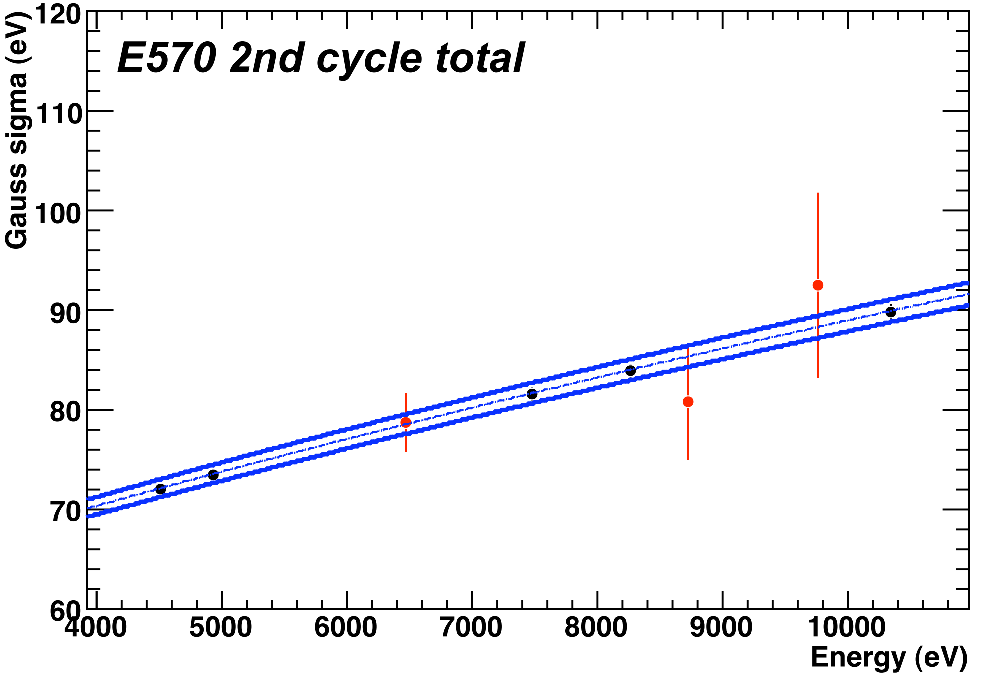


$$N_- = N - \sigma_N$$
$$F_- = F - \sigma_F$$

2007 Aug. 20 Hideyuki Tatsuno

Gaussian sigma (fixed Γ=0)





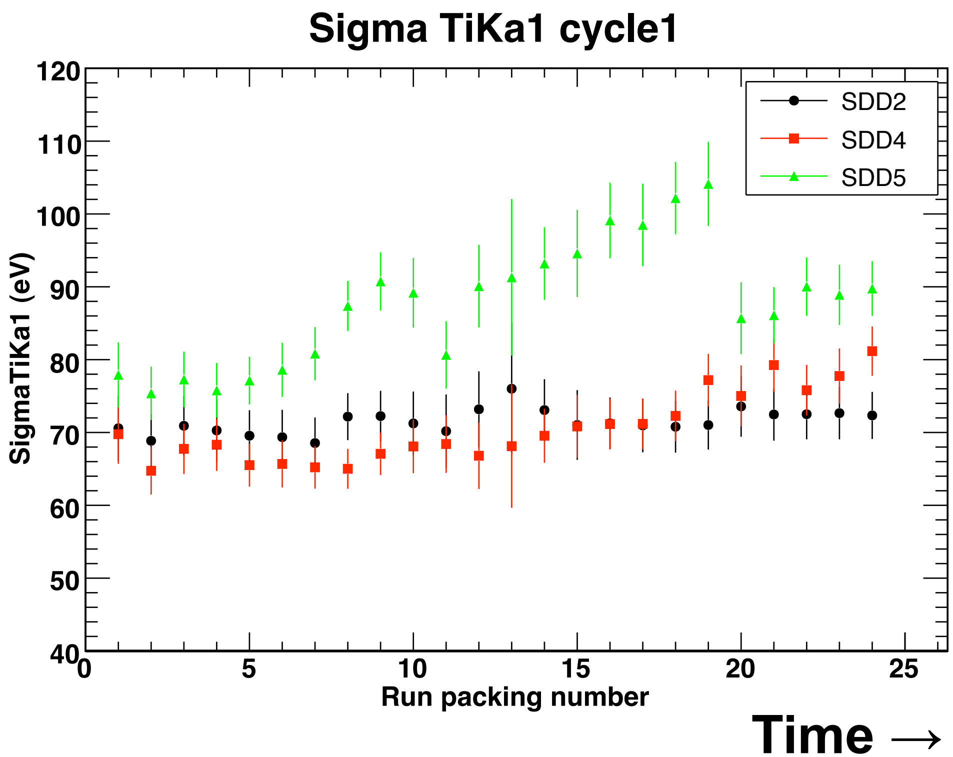
This like difference seems a finite Γ !

But it is not clear the resolution function is accurate or not.

➡ Here I checked how good the summed-up resolution function was determined.

Simulation to check the resolution

Input : real data (time dependent)



Resolution function (sigma)

$$\sigma(N_i, F_i, E, t_i)$$

its error at t_i $\sigma_i(N_i, F_i, E, t_i)$

Fill a dummy histogram with

$$\mathrm{Gaus}(E, \sigma(t_i))$$

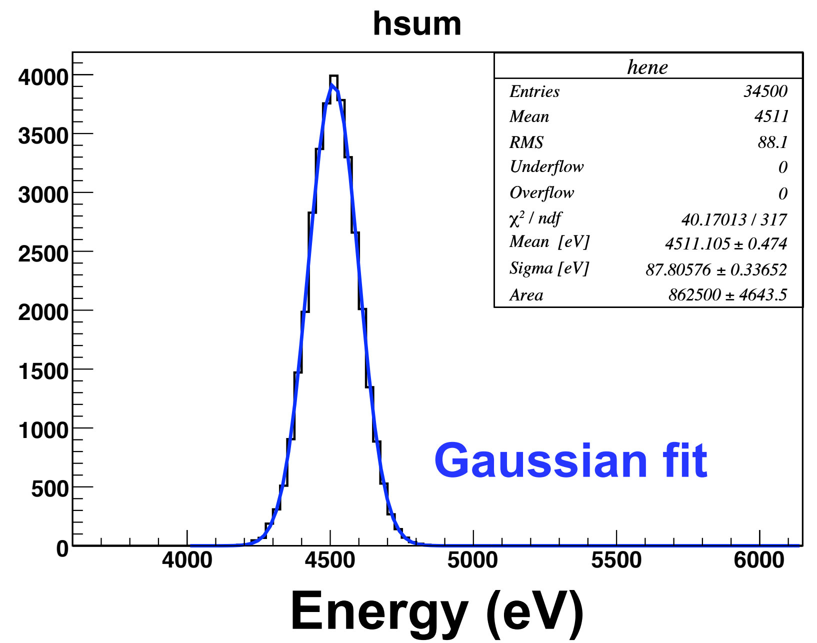
Summed up all histograms by time

$$\sum_{t_i} \mathrm{Gaus}(E, \sigma(t_i))$$

For example, 1st cycle SDD5 (TiKα1)

```
root [1] simple(1500,1,5,1,4510.89)
../input_file/resol/resol-c1-sdd5.param: Initialized
mean  = 4.51089e+03 +- 0.00000e+00
sigma = 8.68156e+01 +- 9.48764e-01

 FCN=269.155 FROM MINOS     STATUS=SUCCESSFUL     62 CALLS          131 TOTAL
                     EDM=7.63091e-09    STRATEGY= 1      ERROR MATRIX ACCURATE
  EXT PARAMETER                PARABOLIC          MINOS ERRORS
  NO.   NAME      VALUE          ERROR      NEGATIVE      POSITIVE
   1  Mean  [eV]   4.51111e+03   4.74324e-01  -4.74328e-01   4.74328e-01
   2  Sigma [eV]   8.78058e+01   3.36525e-01  -3.35481e-01   3.37566e-01
   3  Gamma [eV]   0.00000e+00     fixed
   4  r            4.00000e+00     fixed
   5  Area         8.62500e+05   4.64305e+03  -4.63520e+03   4.65186e+03
```



$\Delta E = +0.22 \pm 0.47$ eV

$\Delta\sigma = +1.0 \pm 0.3$ eV from calc. with Noise and Fano of self-triggered data

Q. Where are the center values of ΔE and $\Delta\sigma$?

Simulated 10K times ..... the converged values are

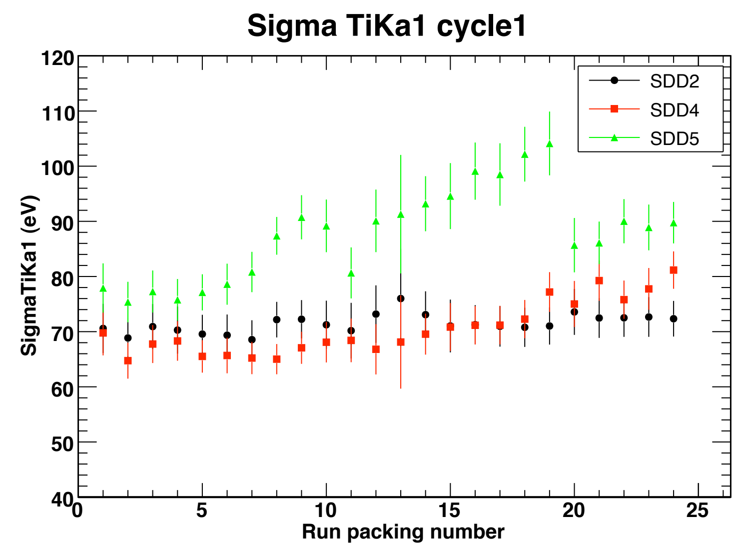
A. $\Delta E = \pm 0.00$ eV, stat. error is ± 0.47 eV

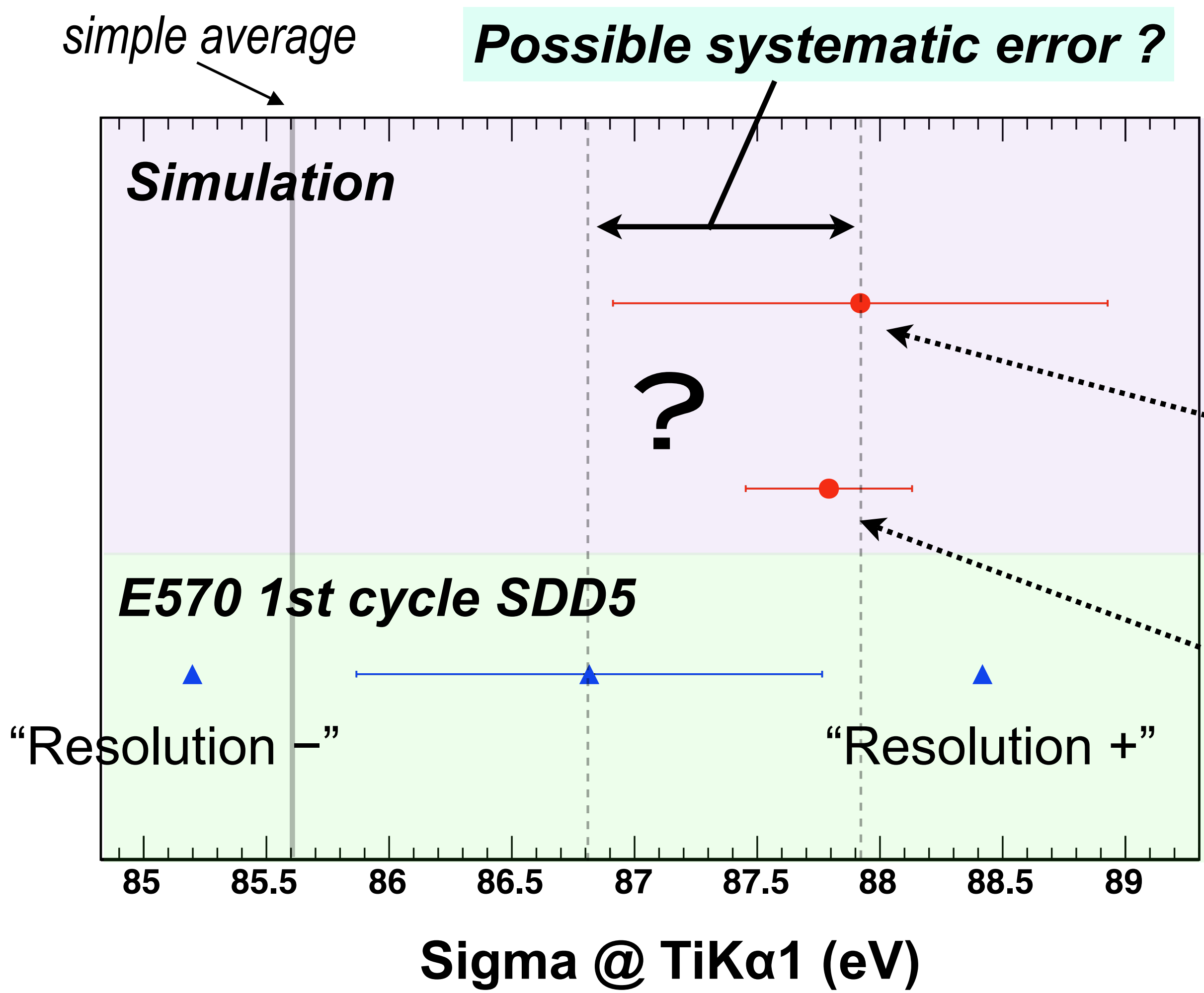
$\Delta\sigma = +1.0$ eV, stat. error is $\pm$ 0.3 eV

?

Assumed their probability density functions are Gaussians

1st cycle SDD5 (TiKα1)





Expected value 1

$$\mathrm{Gaus}(E, \mathrm{Gaus}(\sigma(t_i), \sigma_i))$$

σ with its error

Expected value 2

$$\mathrm{Gaus}(E, \sigma(t_i))$$

center value of σ

Why is the simulated sigma larger !?

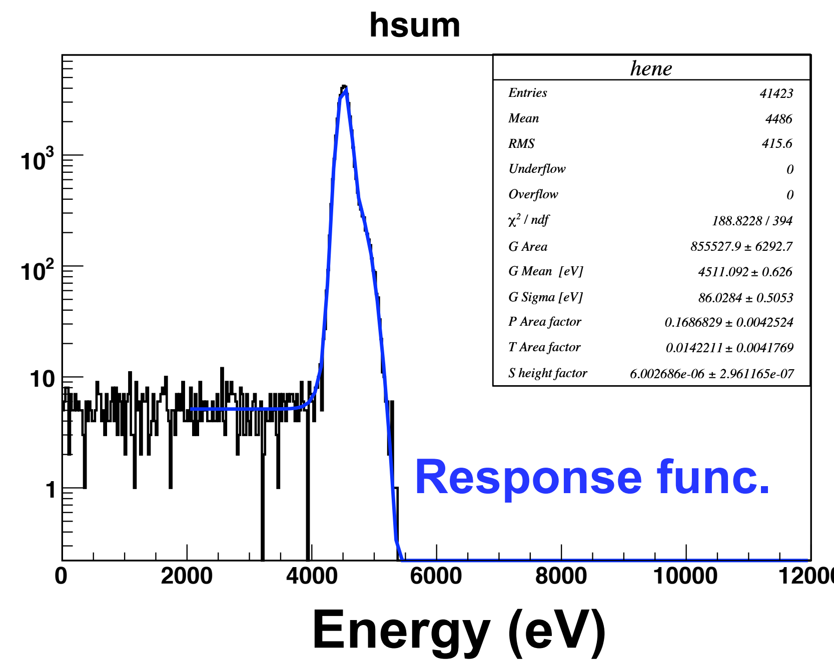
For example, 1st cycle SDD5 (TiKα1)

Simple Gaussian → Response function

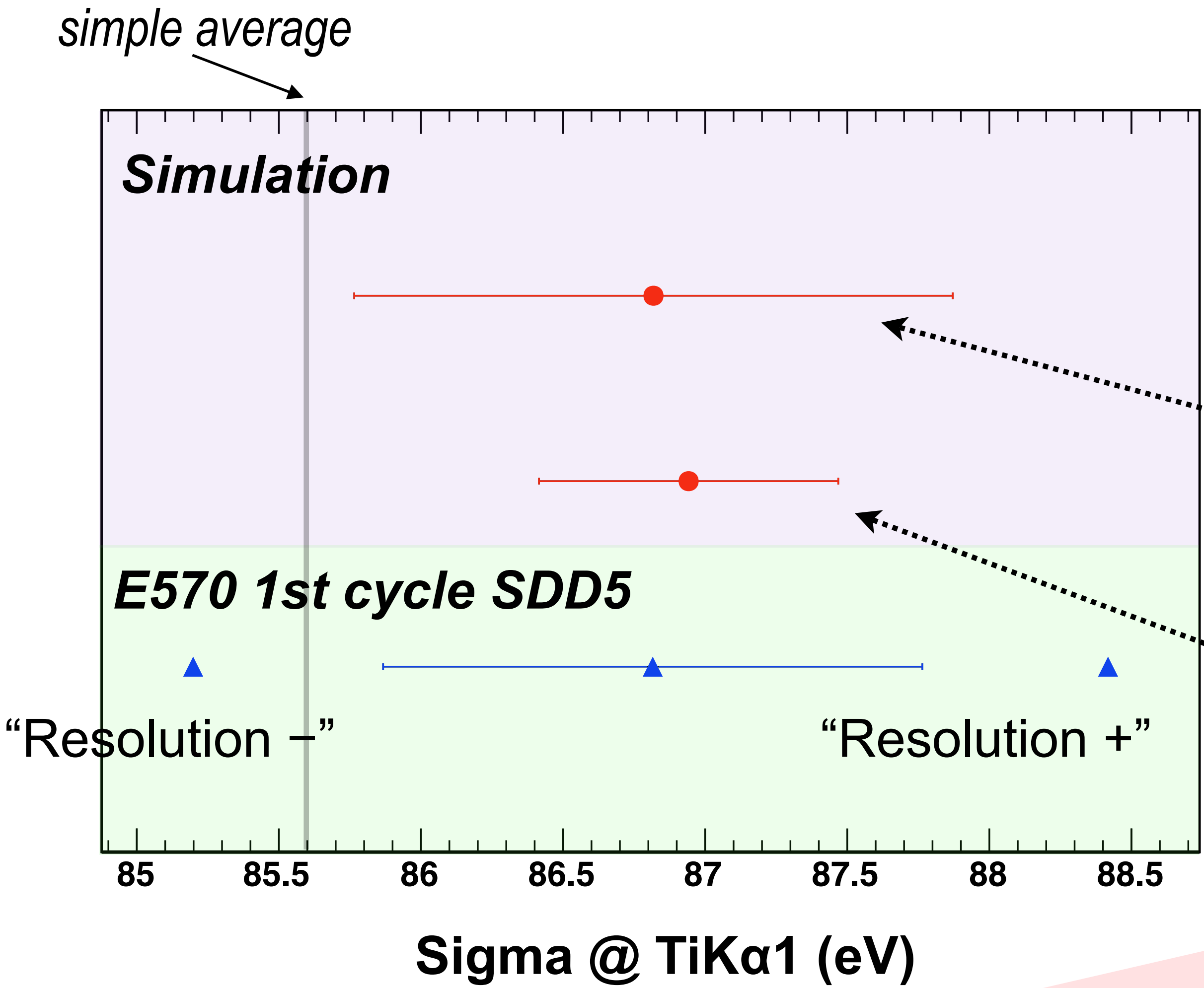
more realistic

```
mean  = 4.51089e+03 +- 0.00000e+00
sigma = 8.68156e+01 +- 9.48764e-01
sigma (resolution minus) = 8.51984e+01
sigma (resolution plus)  = 8.84177e+01

 FCN=758.158 FROM MINOS     STATUS=SUCCESSFUL    413 CALLS         599 TOTAL
                     EDM=7.7025e-06    STRATEGY= 1      ERROR MATRIX ACCURATE
  EXT PARAMETER                  PARABOLIC         MINOS ERRORS
  NO.   NAME      VALUE            ERROR      NEGATIVE      POSITIVE
   1  G Area       8.55528e+05   6.29203e+03  -6.31363e+03   6.27169e+03
   2  G Mean  [eV]   4.51109e+03   6.25747e-01  -6.24051e-01   6.27491e-01
   3  G Sigma [eV]   8.60284e+01   5.05298e-01  -5.05171e-01   5.05503e-01
   4  P Area factor   1.68683e-01   4.25217e-03  -4.21801e-03   4.28677e-03
   5  P Shift [eV]   2.00000e+02      fixed
   6  P Sigma factor   2.00000e+00      fixed
   7  T Area factor   1.42211e-02   4.17376e-03  -4.04139e-03   4.31244e-03
   8  T Slope beta   1.20000e+00      fixed
   9  S height factor   6.00269e-06   2.96099e-07  -2.91453e-07   3.00780e-07
```

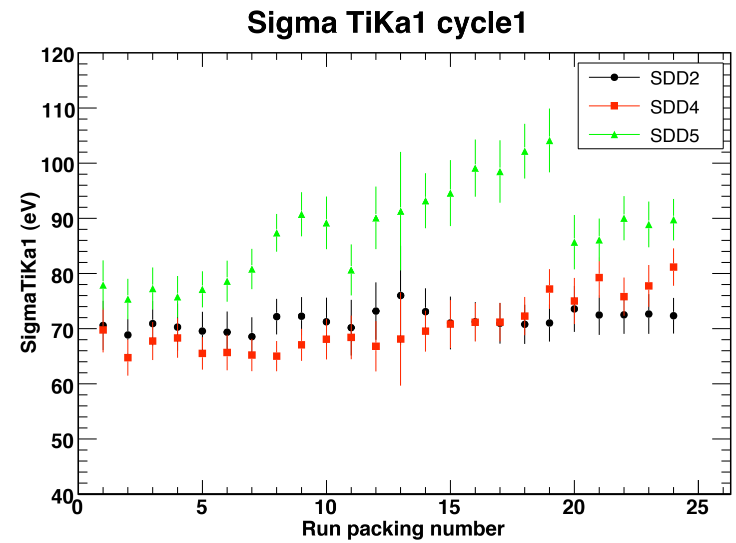


1st cycle SDD5 (TiKα1) Response function



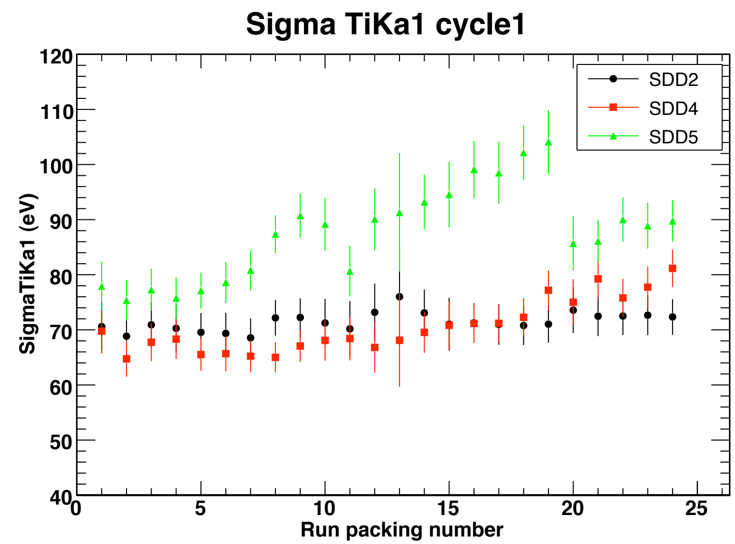
Expected value 1
σ with its error

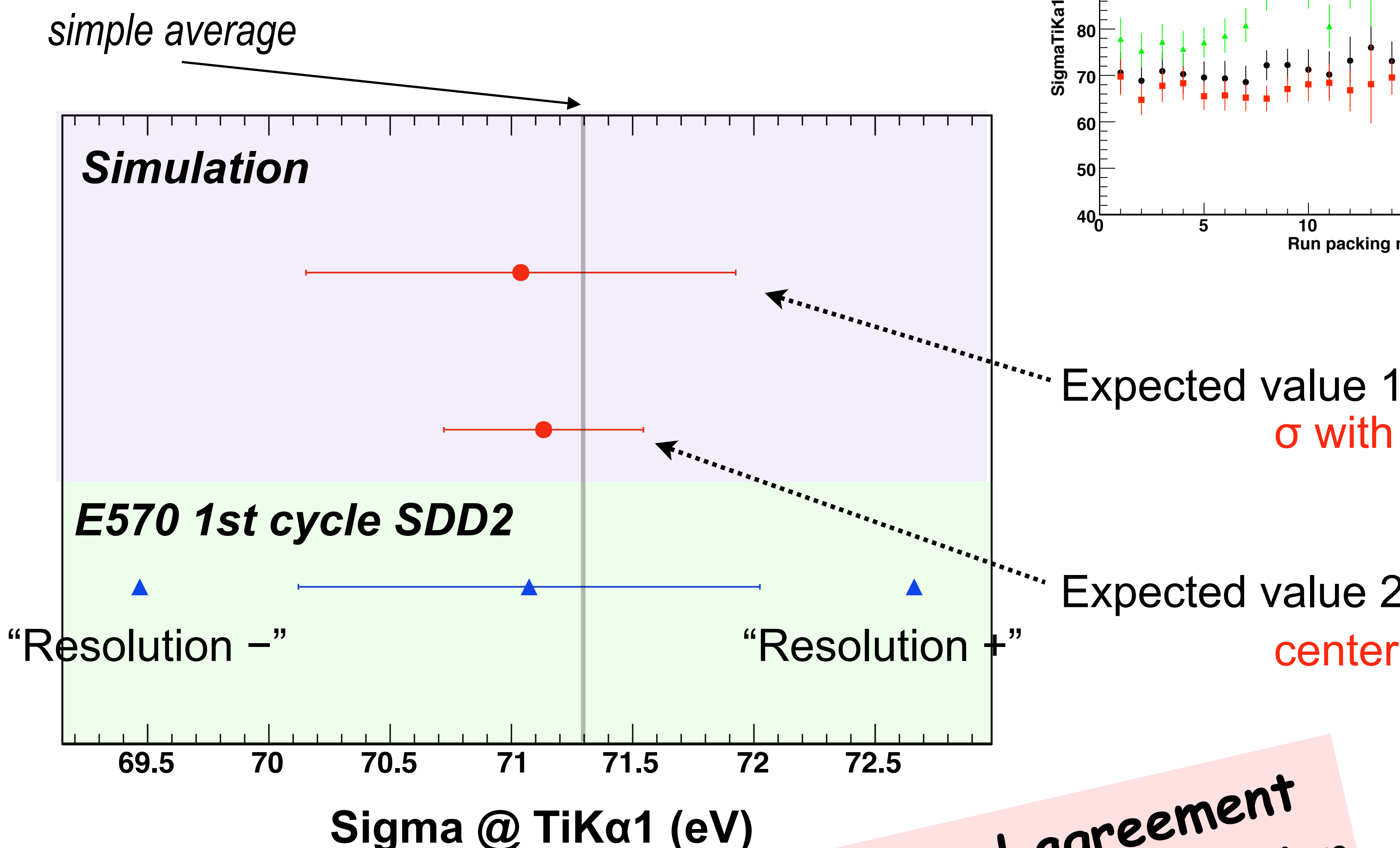
Expected value 2
center value of σ



Good agreement with the simulation

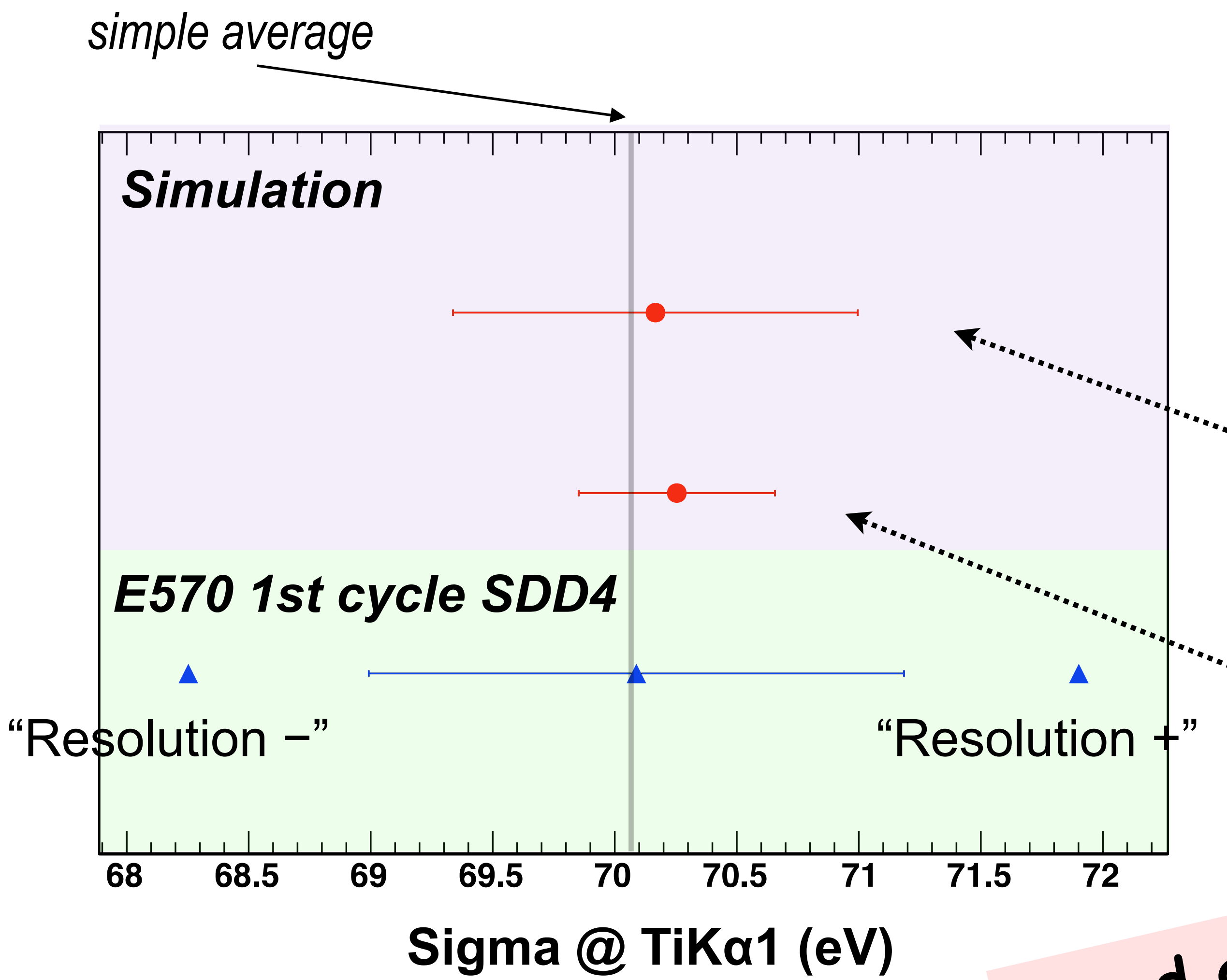
1st cycle SDD2 (TiKα1) Response function

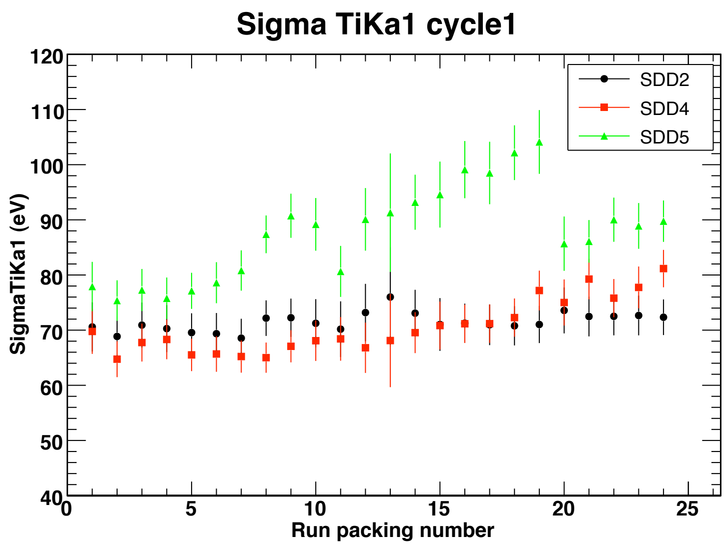




Good agreement with the simulation

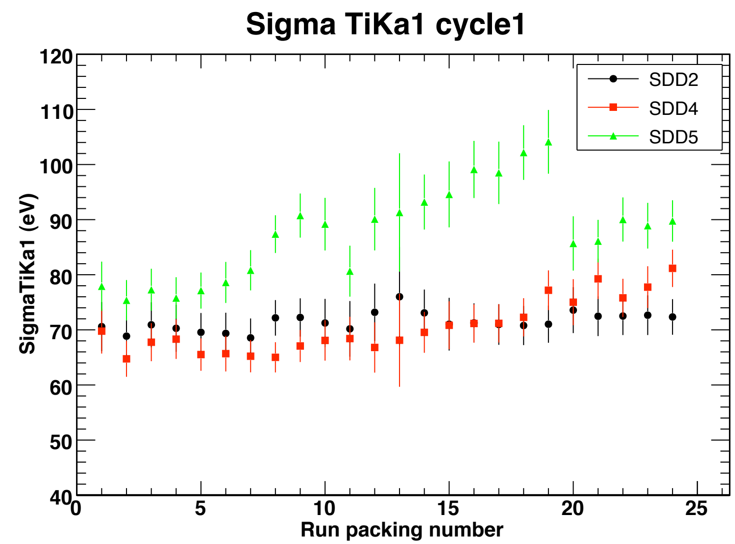
1st cycle SDD4 (TiKα1) Response function





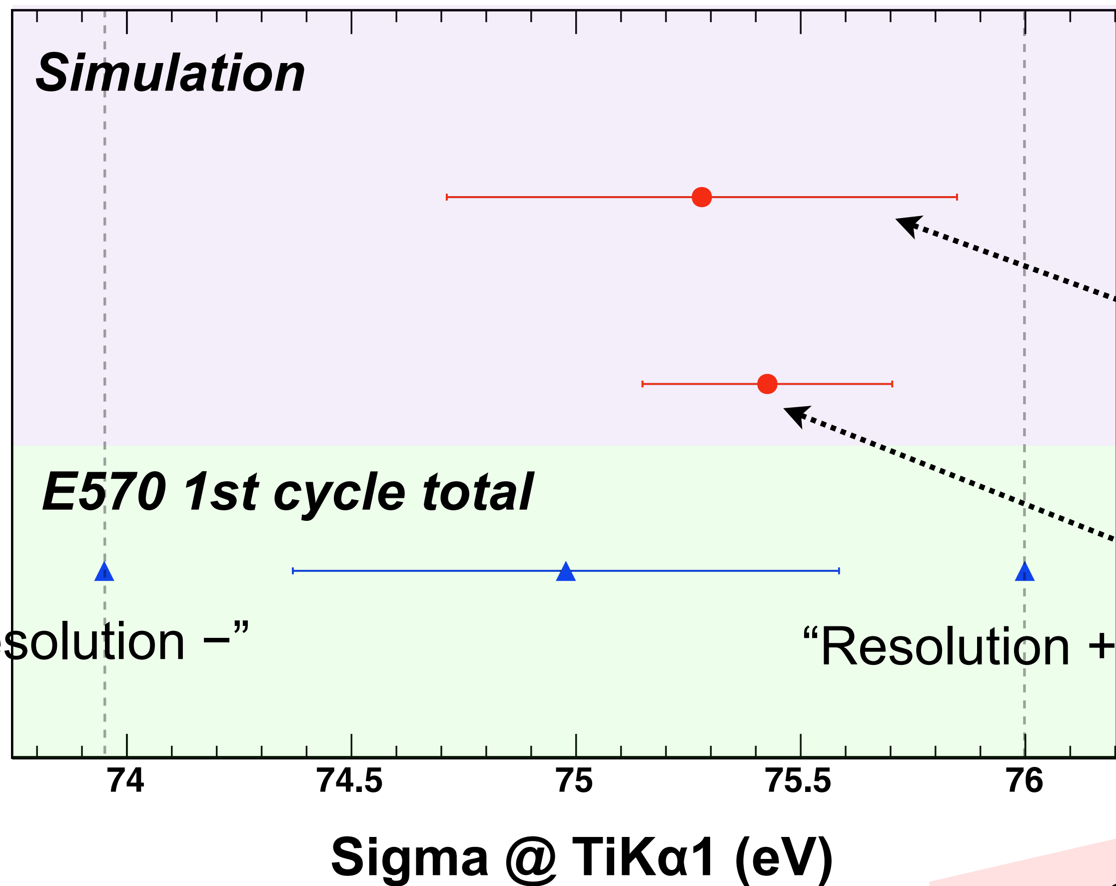
Good agreement with the simulation

1st cycle total (TiKα1)
Response function



Lower limit

Upper limit



“Resolution −”

“Resolution +”

Expected value 1

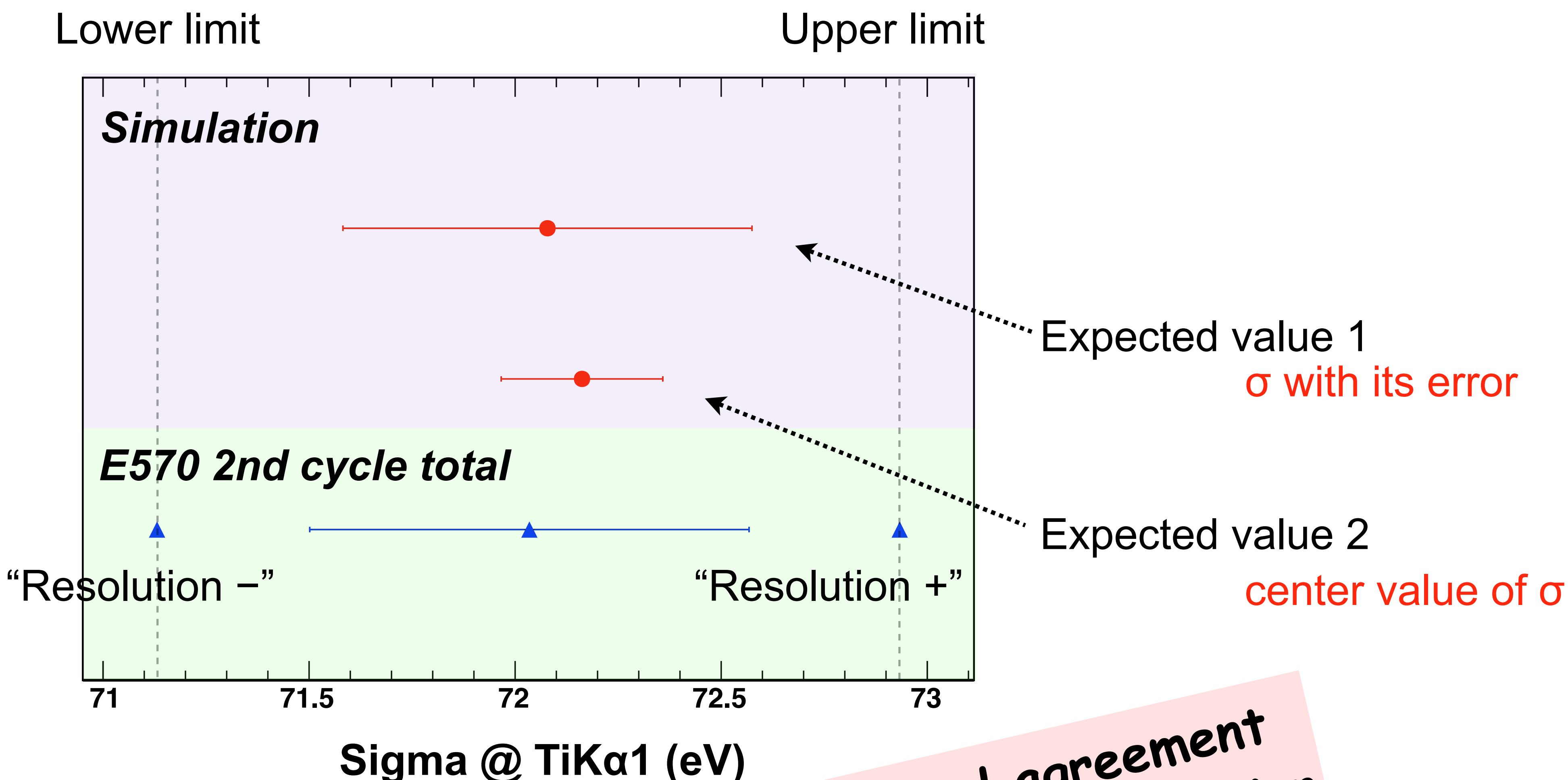
σ with its error

Expected value 2

center value of σ

Good agreement with the simulation

2nd cycle total (TiKα1)
Response function



Good agreement with the simulation

Summary

The summed-up resolution was reproduced well by a simulation using a response function (not good by simple Gaussian)

“Resolution plus/minus” are conservative limits to estimate a systematic error of Γ

The upper limit of Γ can be determined from “Resolution minus”

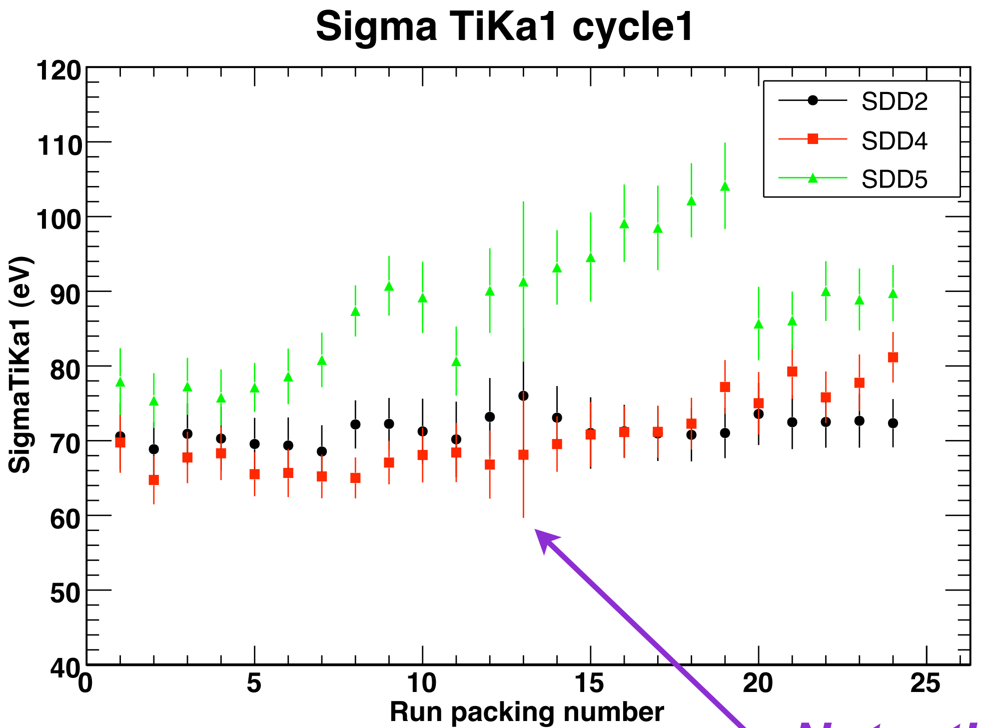
P.S.

Clearly the resolution minus is too much !

It’s better to check the relation between σ and Γ, but this is difficult (the unified fit does not converge...)

Backups

Quality plots



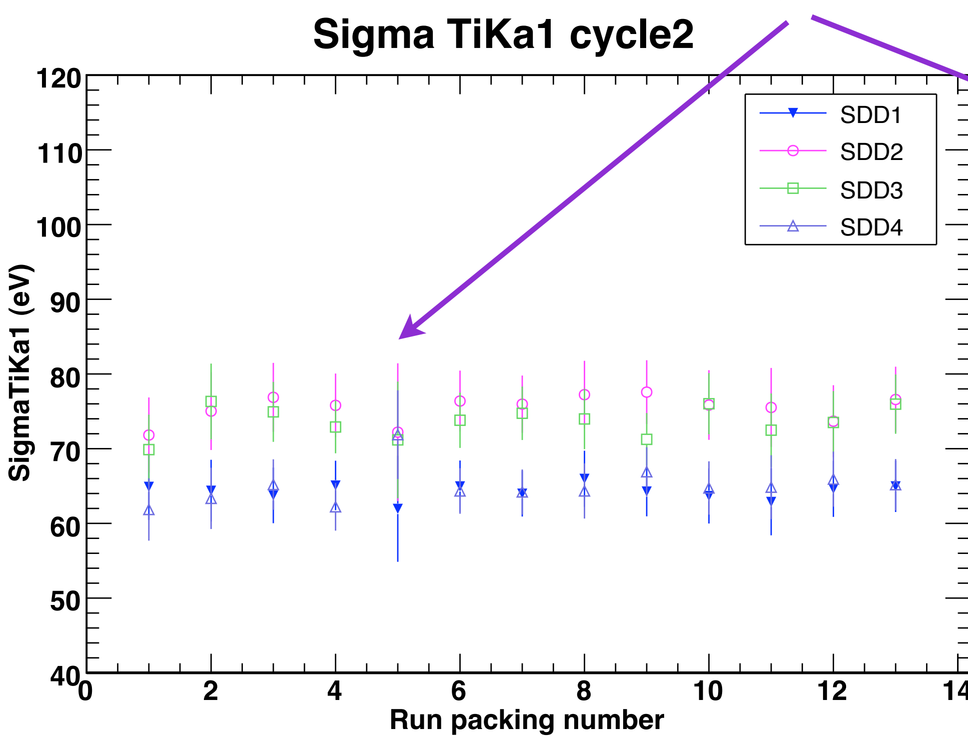
Quality plot (Sigma)

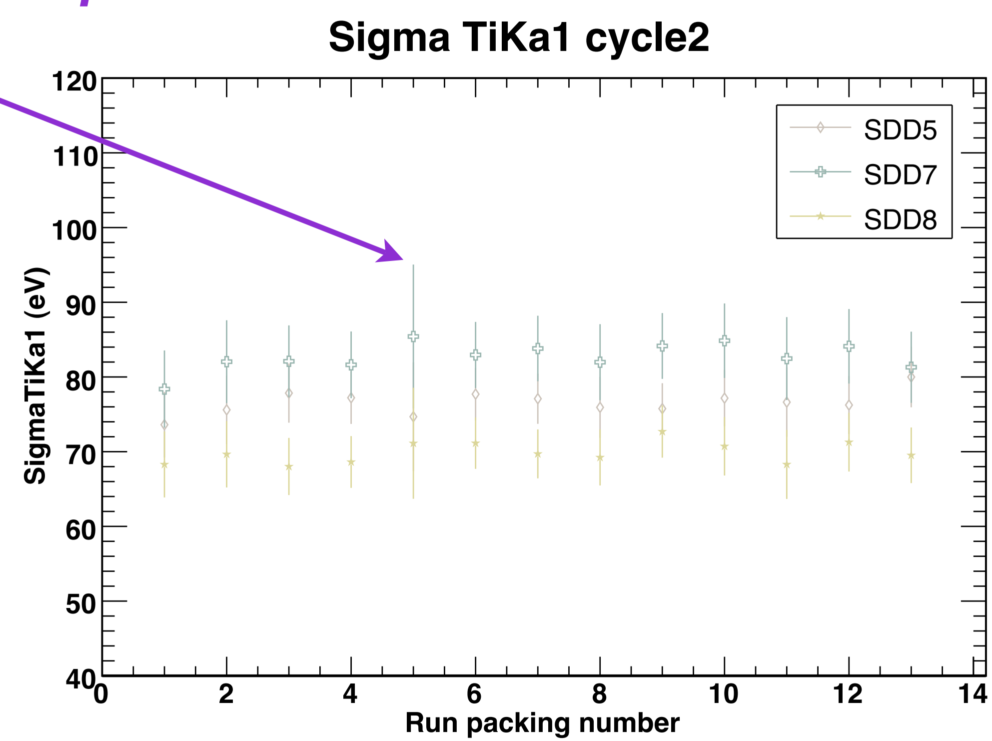
SDD-by-SDD

Sigma was calculated by

$$\sigma = \sqrt{N^2 + FwE}$$

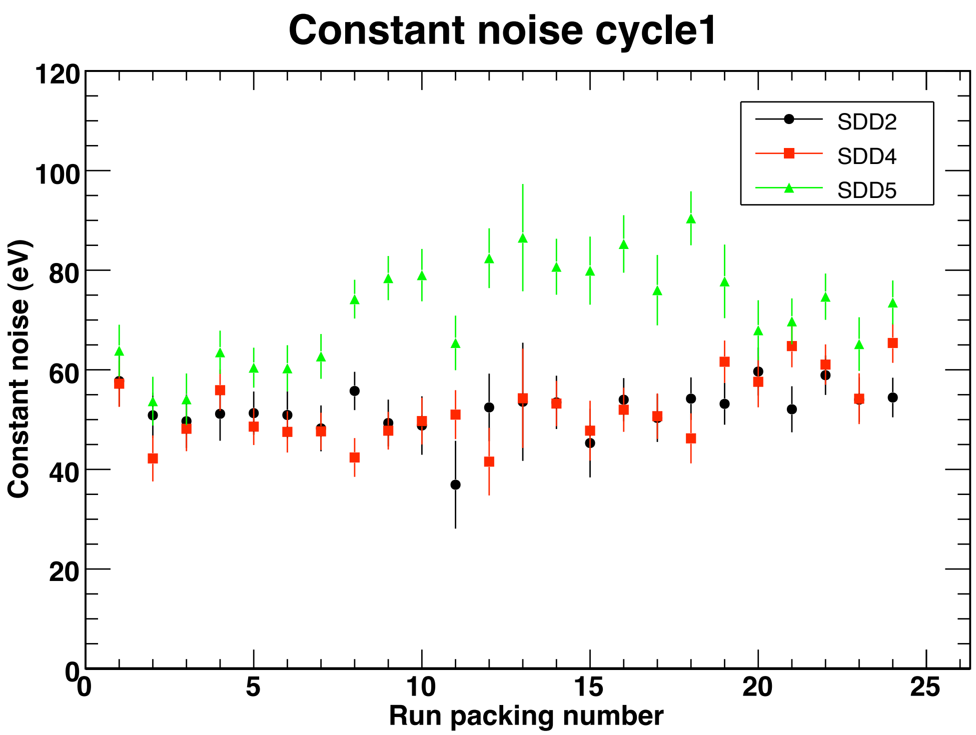
Note: these points were removed

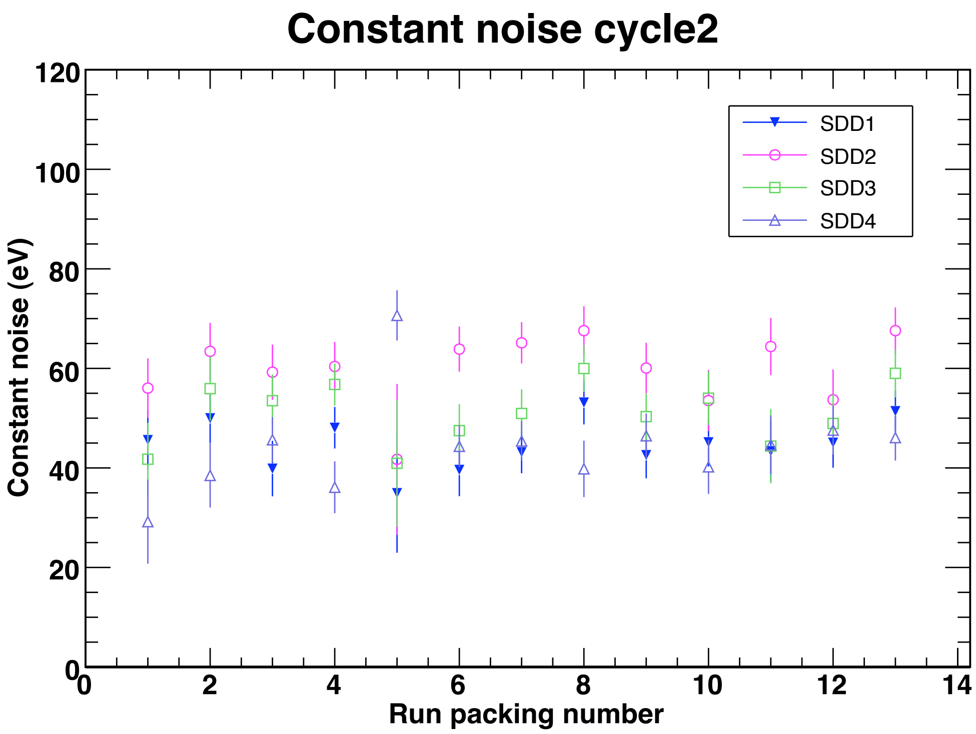


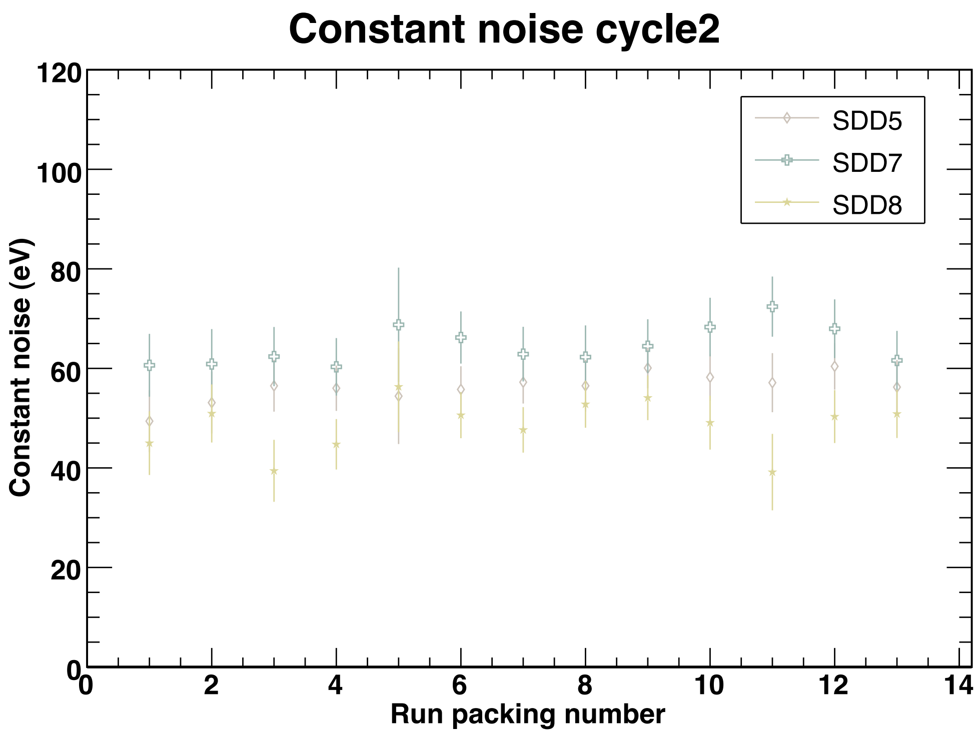


Quality plot (Noise)

SDD-by-SDD

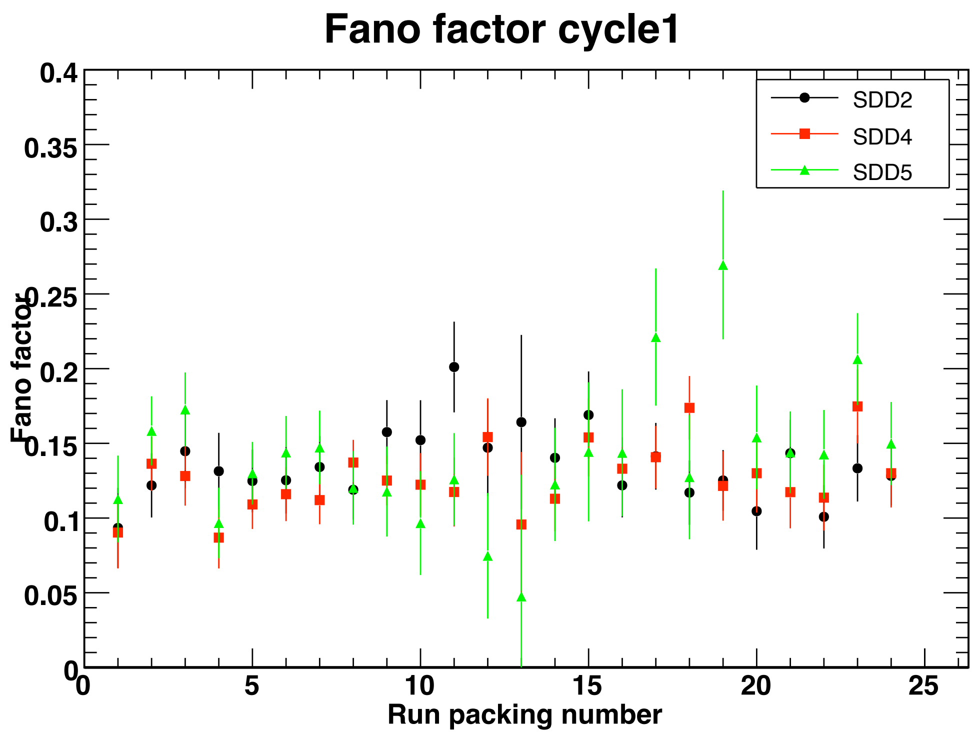


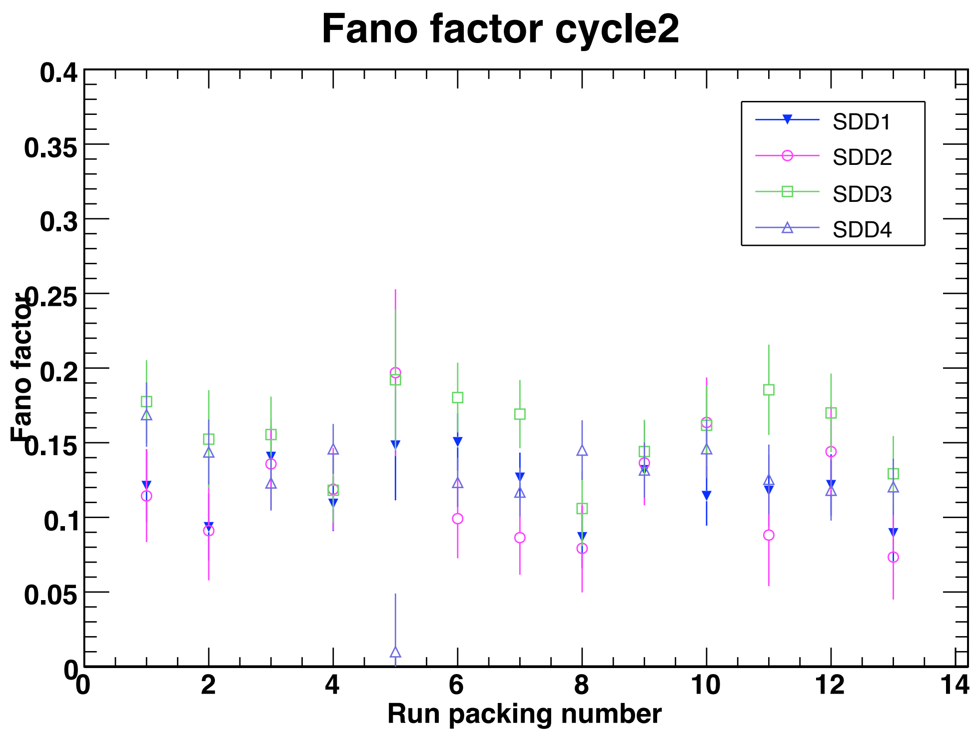


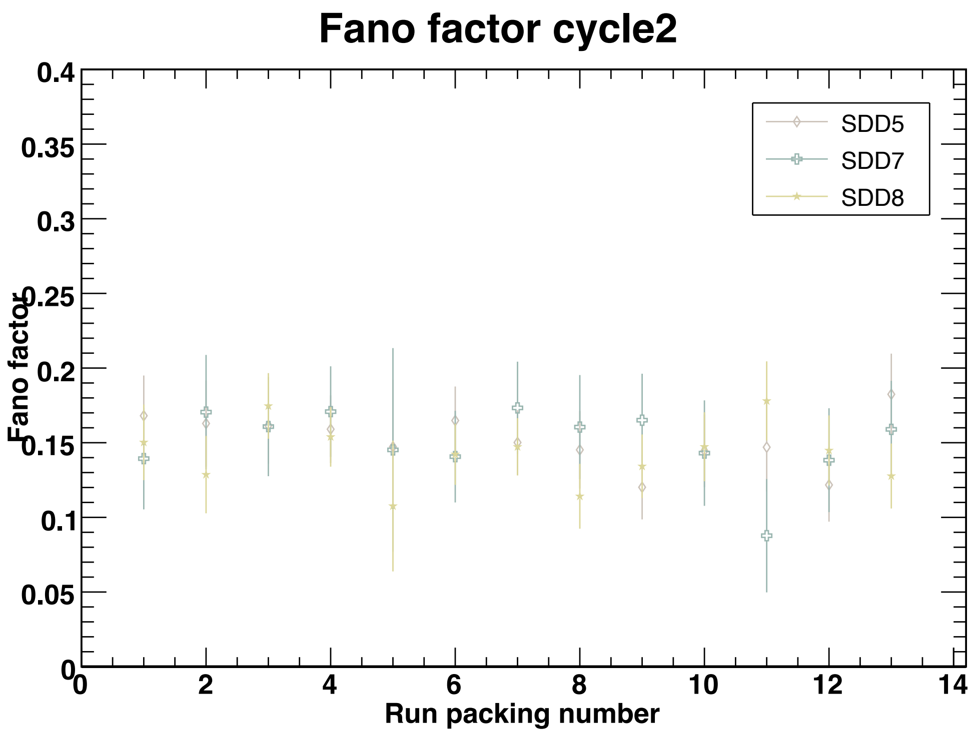


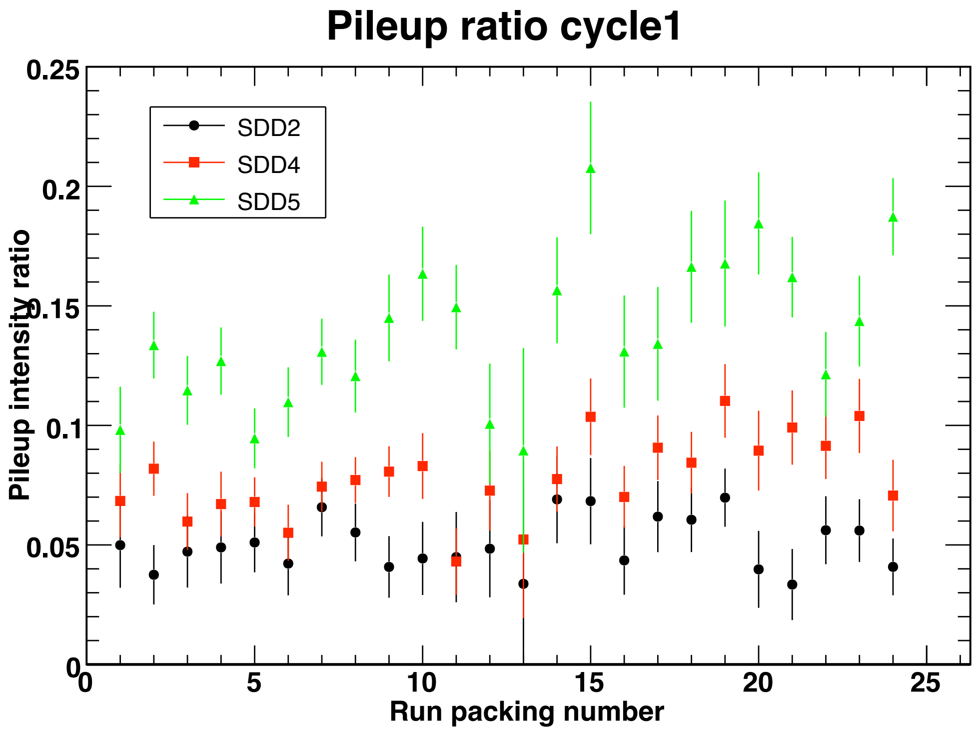
Quality plot (Fano)

SDD-by-SDD



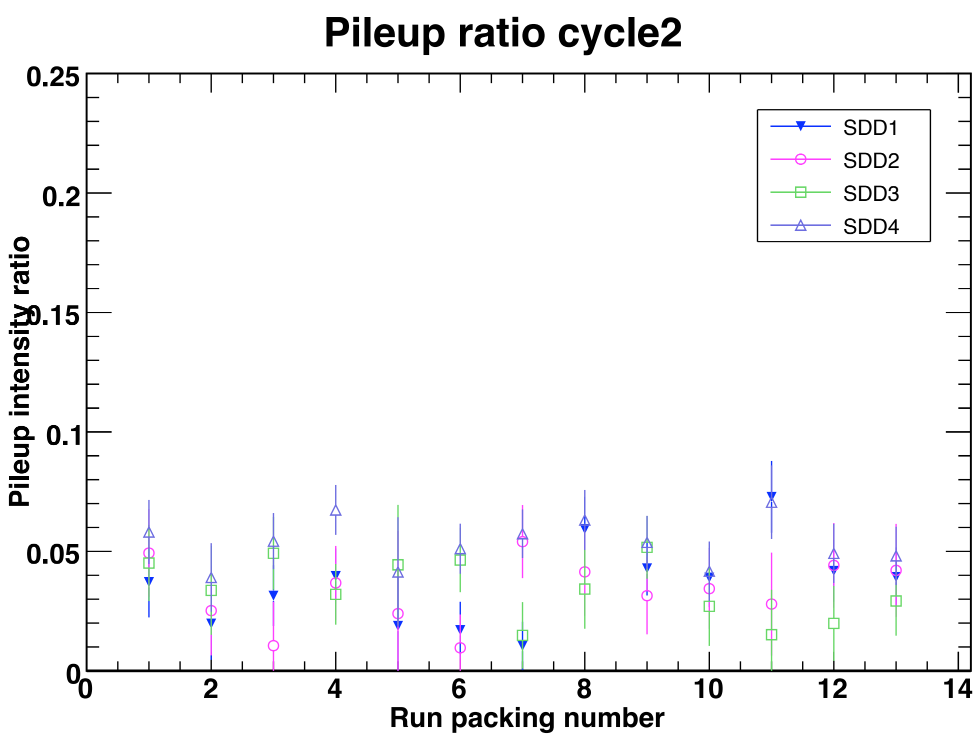


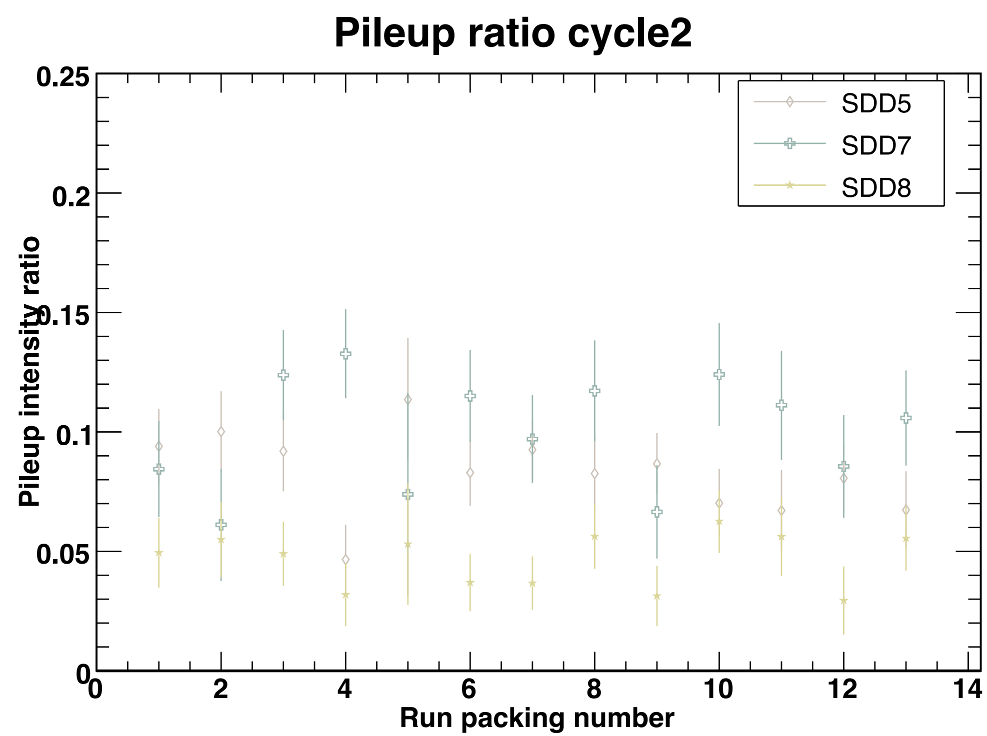




Quality plot (Pileup ratio)

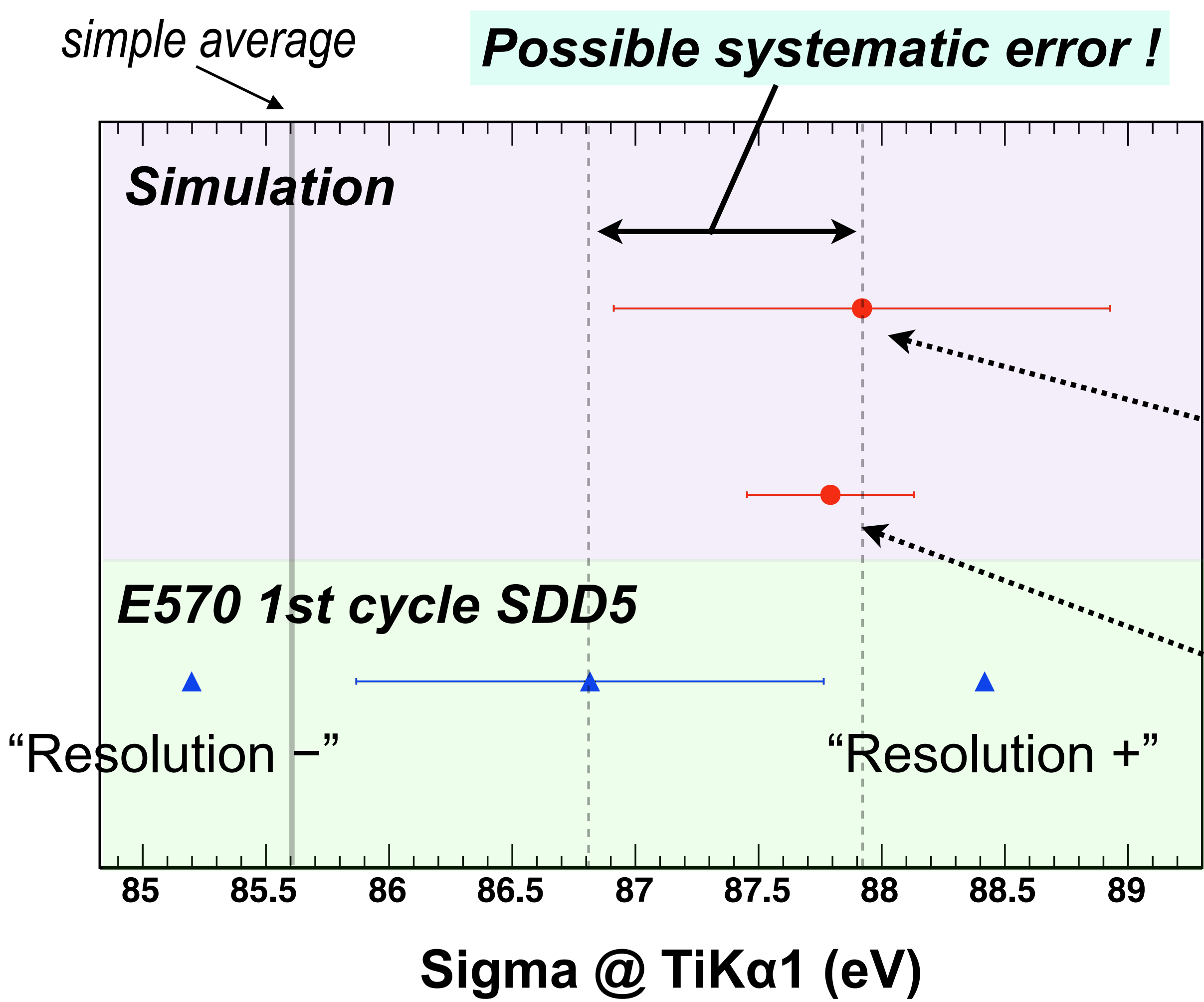
SDD-by-SDD

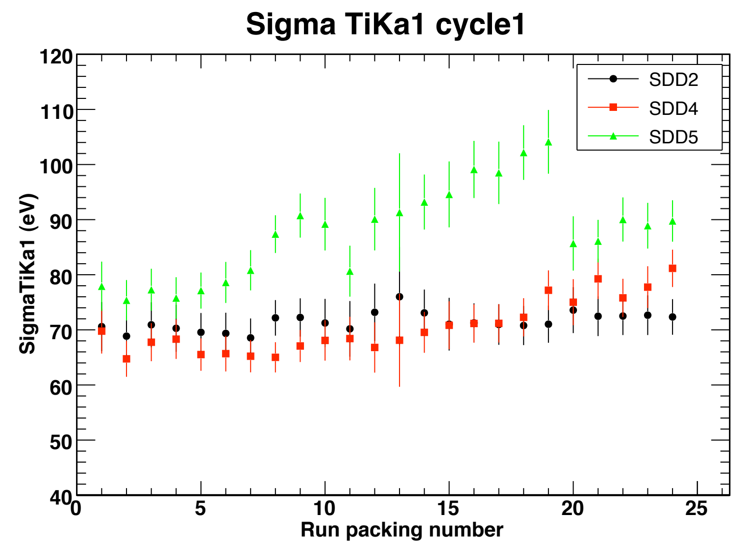




Simple Gaussian simulation
(Not realistic one)

1st cycle SDD5 (TiKα1)





Expected value 1

$$\mathrm{Gaus}(E, \mathrm{Gaus}(\sigma(t_i), \sigma_i))$$

σ with its error

Expected value 2

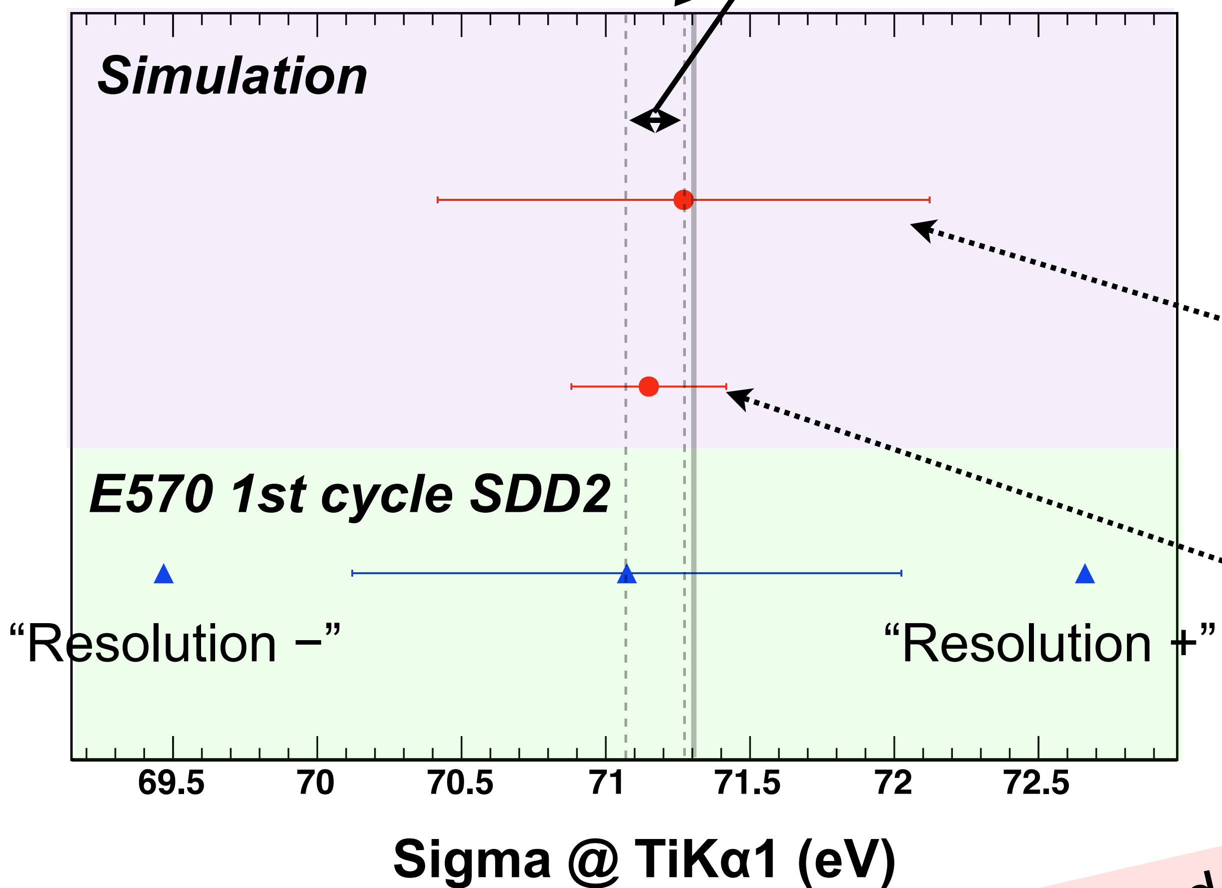
$$\mathrm{Gaus}(E, \sigma(t_i))$$

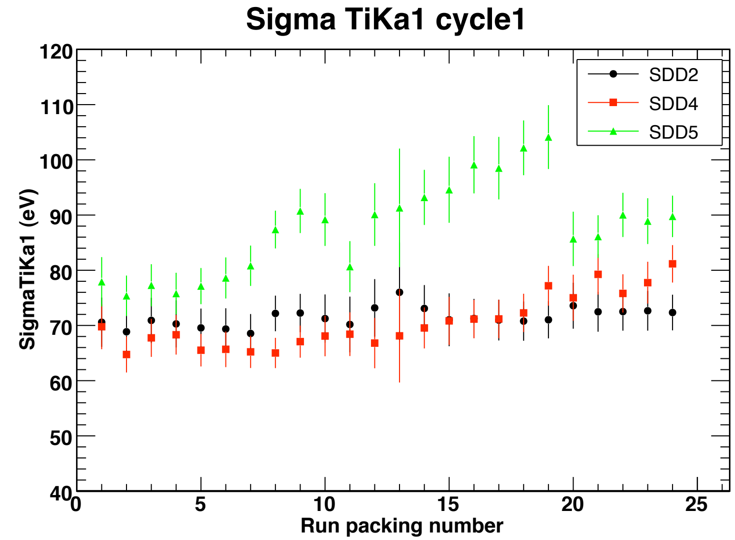
center value of σ

1st cycle SDD2 (TiKα1)

simple average

Possible systematic error !





Expected value 1

$$\mathrm{Gaus}(E, \mathrm{Gaus}(\sigma(t_i), \sigma_i))$$

σ with its error

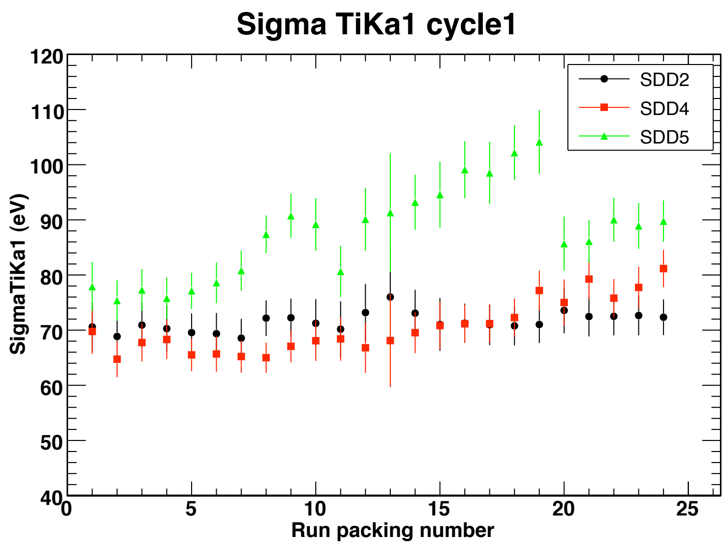
Expected value 2

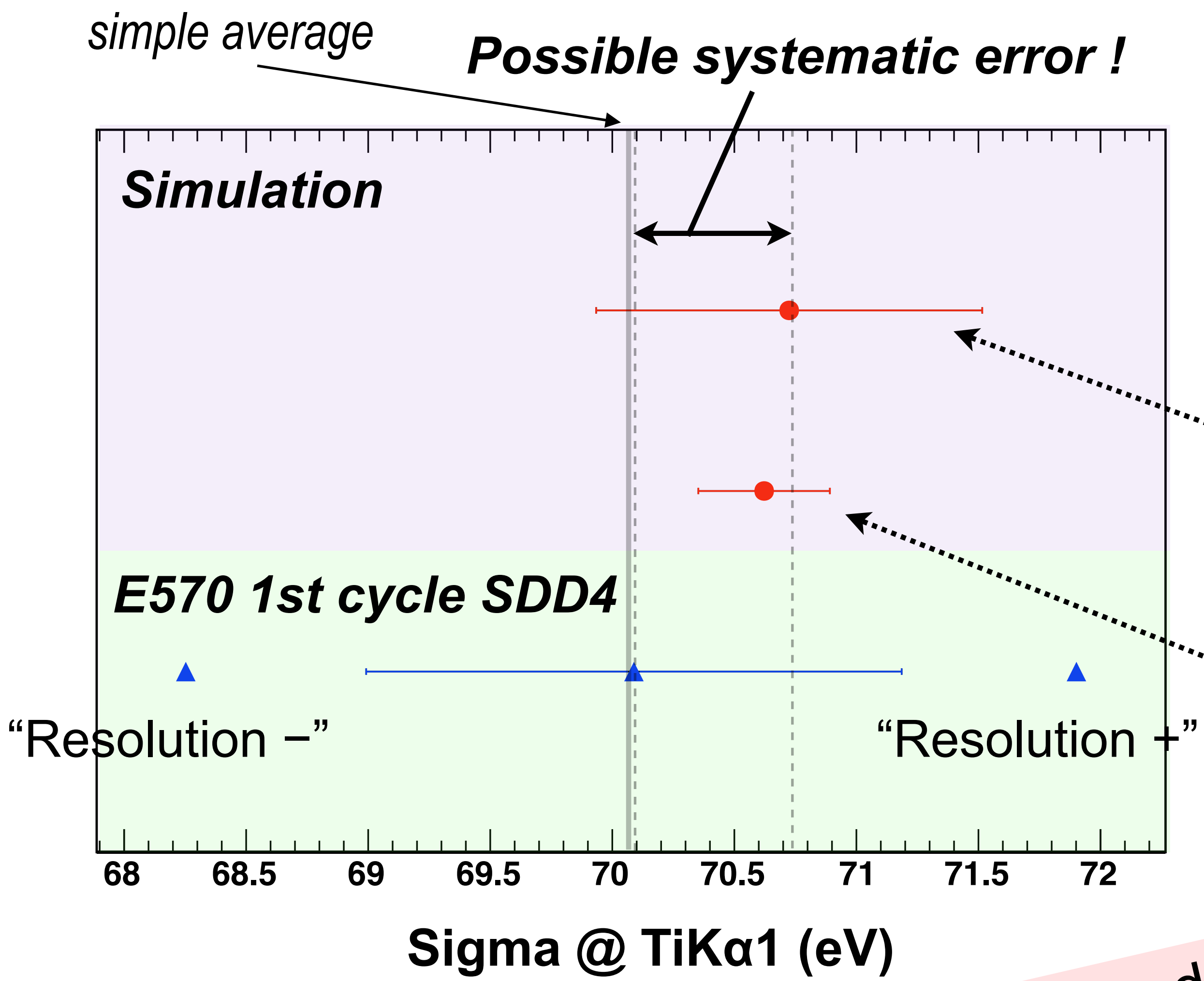
$$\mathrm{Gaus}(E, \sigma(t_i))$$

center value of σ

Good agreement with the simulation

1st cycle SDD4 (TiKα1)





Expected value 1

$$\mathrm{Gaus}(E, \mathrm{Gaus}(\sigma(t_i), \sigma_i))$$

σ with its error

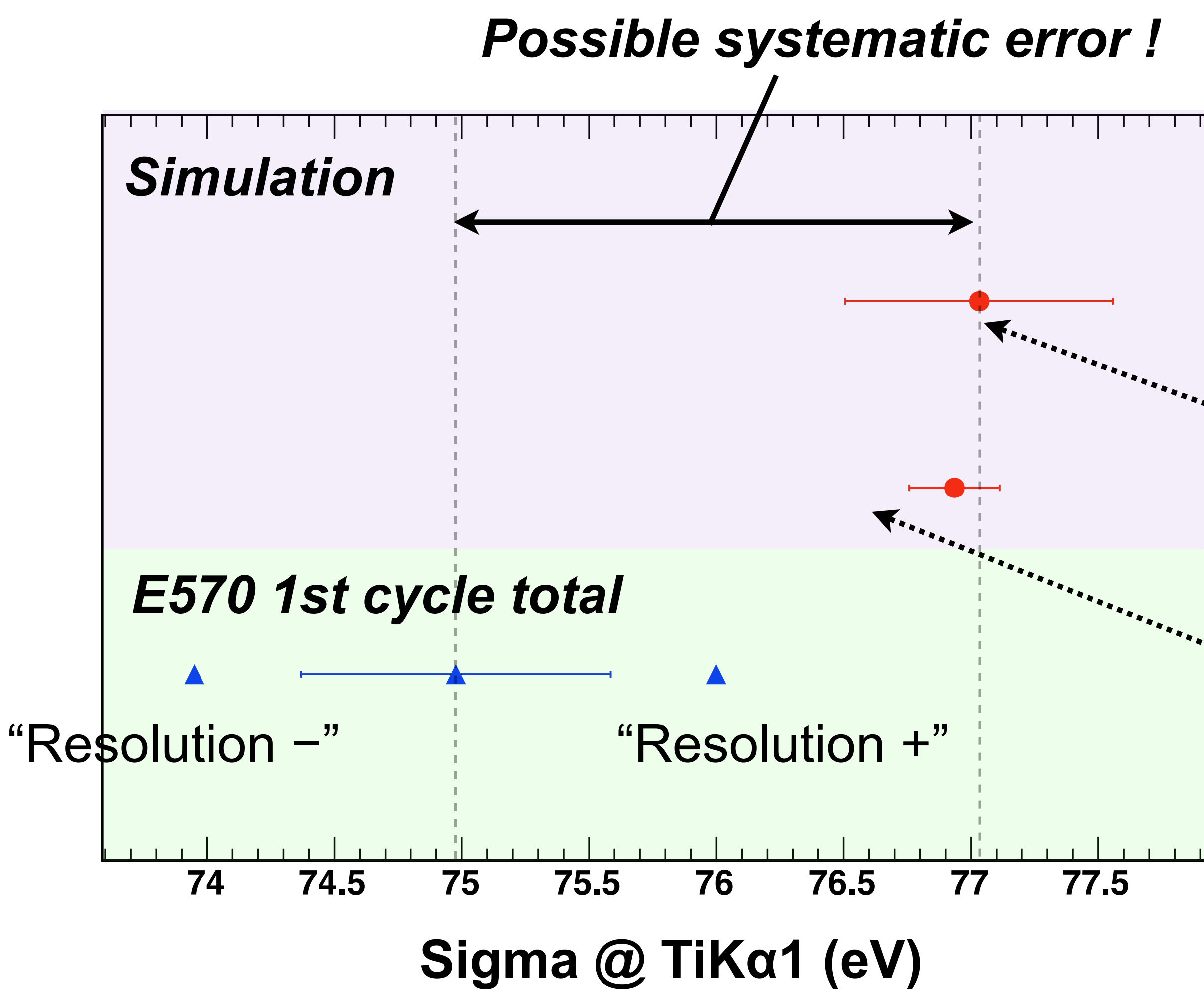
Expected value 2

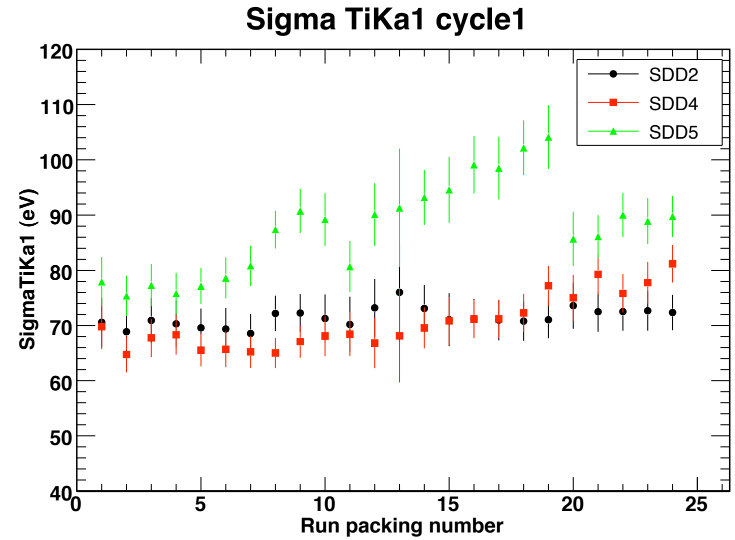
$$\mathrm{Gaus}(E, \sigma(t_i))$$

center value of σ

Good agreement with the simulation

1st cycle total (TiKα1)





Expected value 1

$$\mathrm{Gaus}(E, \mathrm{Gaus}(\sigma(t_i), \sigma_i))$$

σ with its error

Expected value 2

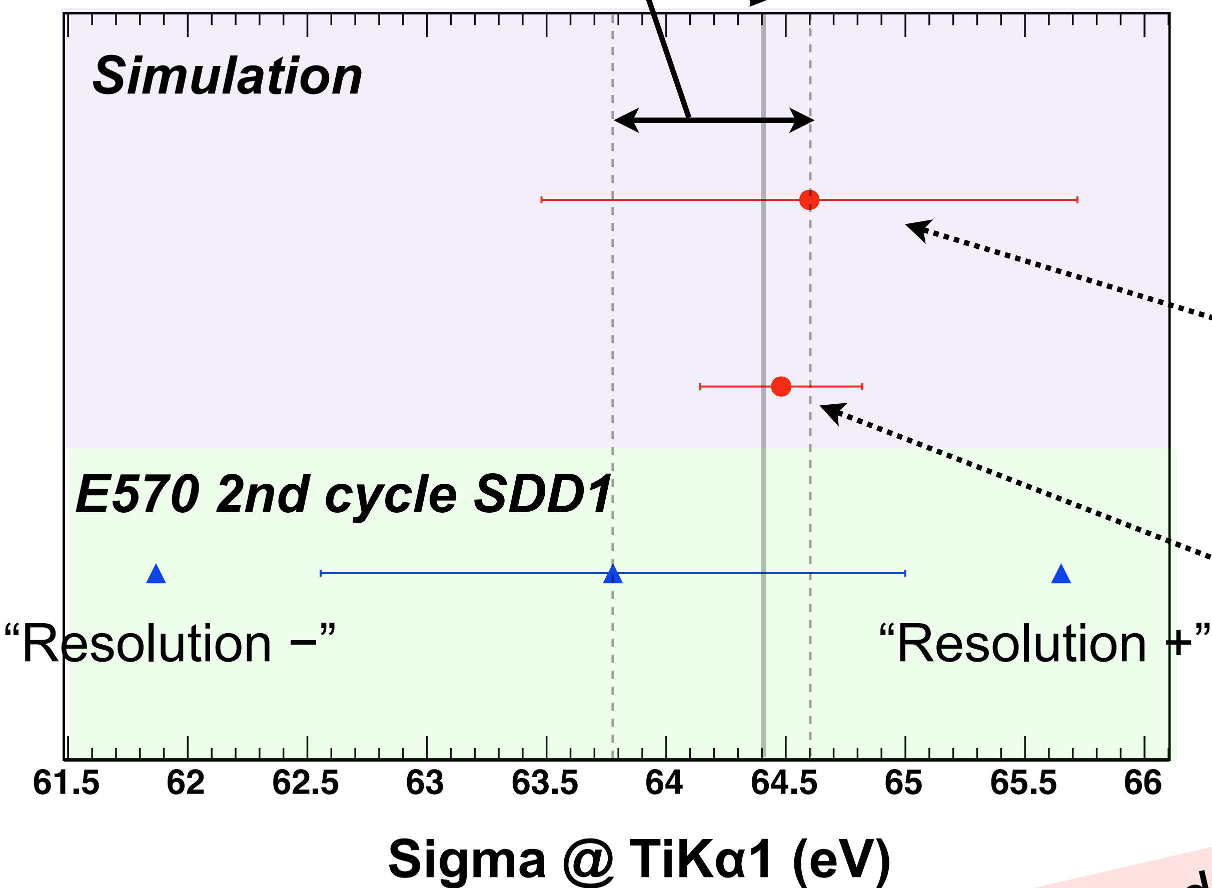
$$\mathrm{Gaus}(E, \sigma(t_i))$$

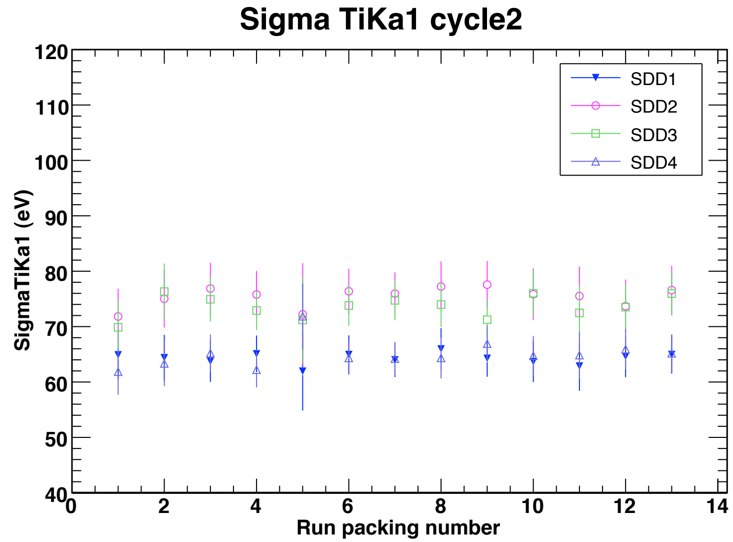
center value of σ

2nd cycle SDD1 (TiKα1)

simple average

Possible systematic error !





Expected value 1

$$\mathrm{Gaus}(E, \mathrm{Gaus}(\sigma(t_i), \sigma_i))$$

σ with its error

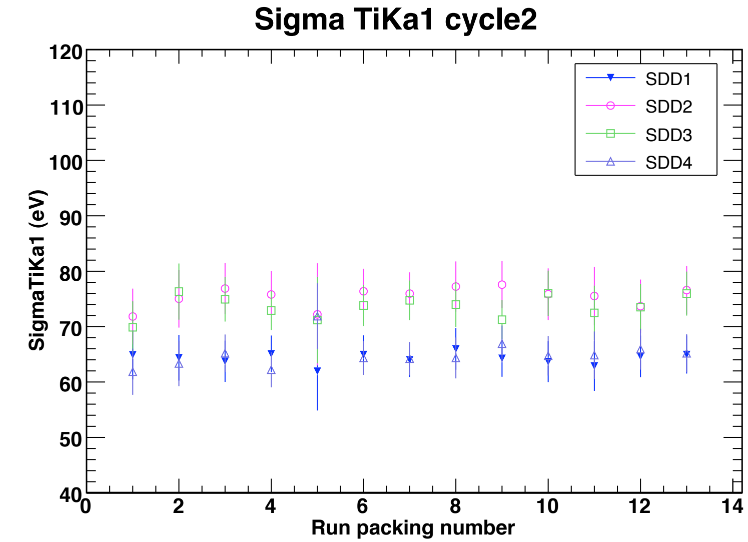
Expected value 2

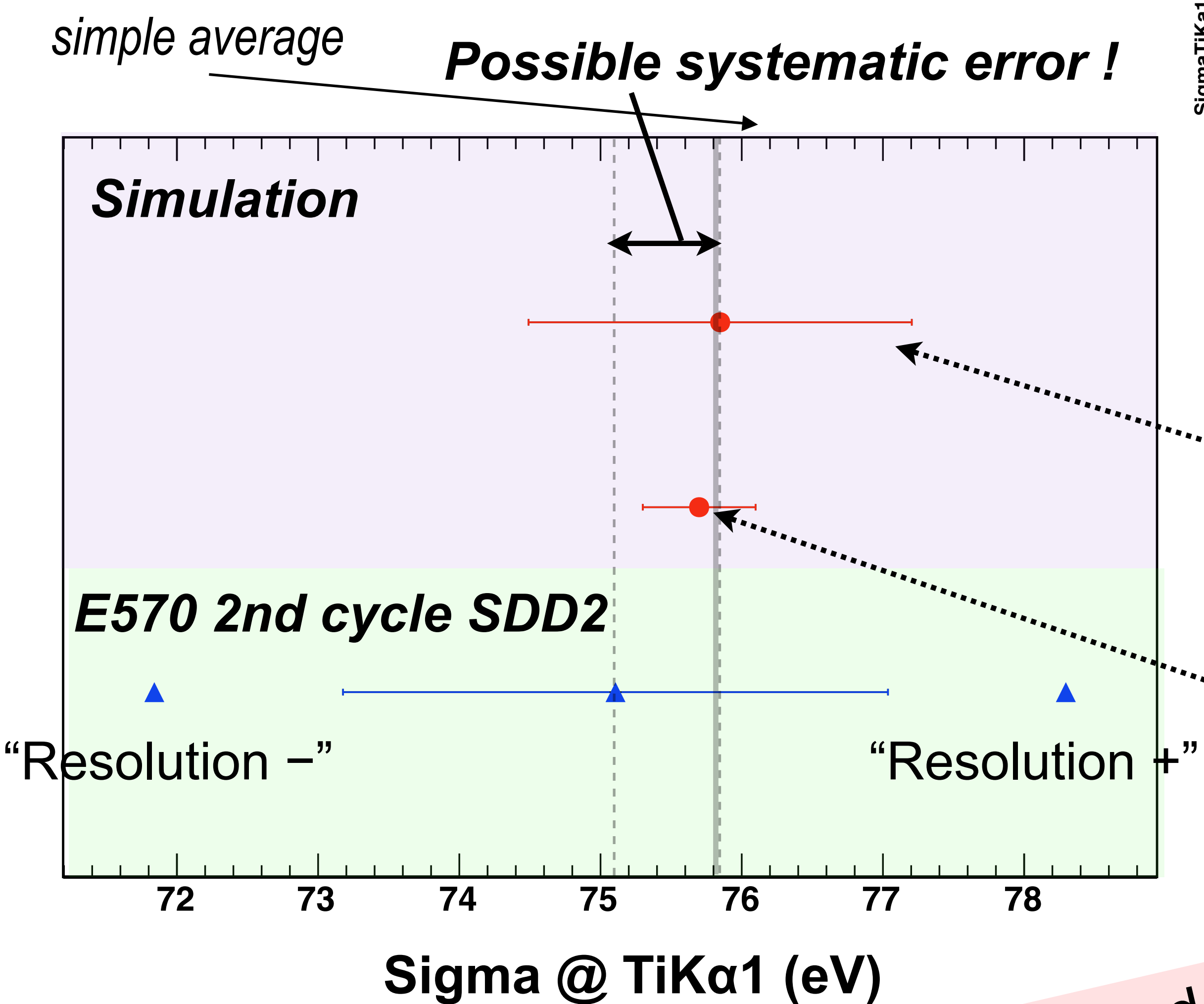
$$\mathrm{Gaus}(E, \sigma(t_i))$$

center value of σ

Good agreement with the simulation

2nd cycle SDD2 (TiKα1)





Expected value 1

$$\mathrm{Gaus}(E, \mathrm{Gaus}(\sigma(t_i), \sigma_i))$$

σ with its error

Expected value 2

$$\mathrm{Gaus}(E, \sigma(t_i))$$

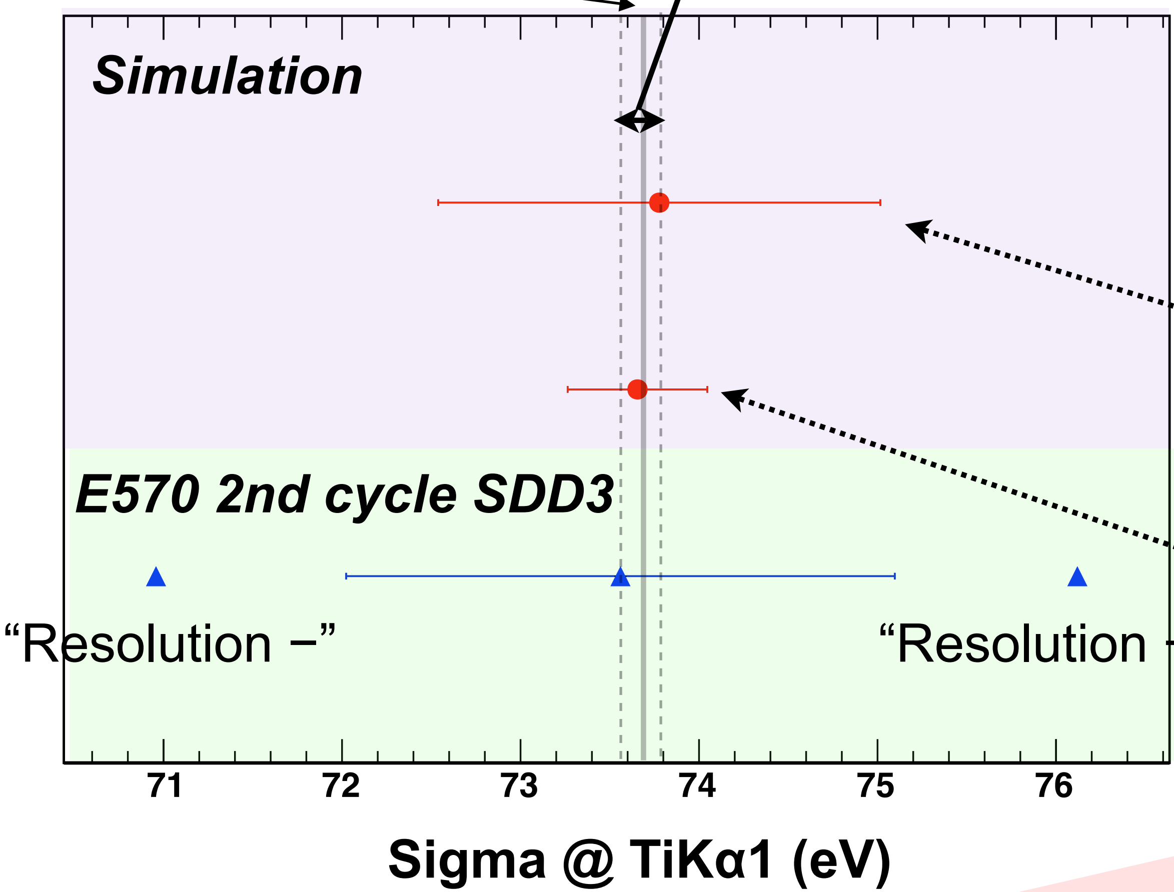
center value of σ

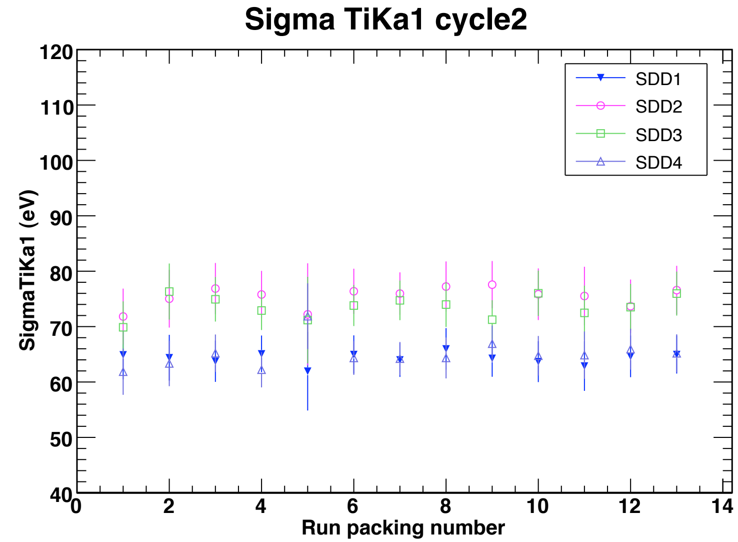
Good agreement with the simulation

2nd cycle SDD3 (TiKα1)

simple average

Possible systematic error !





Expected value 1

$$\mathrm{Gaus}(E, \mathrm{Gaus}(\sigma(t_i), \sigma_i))$$

σ with its error

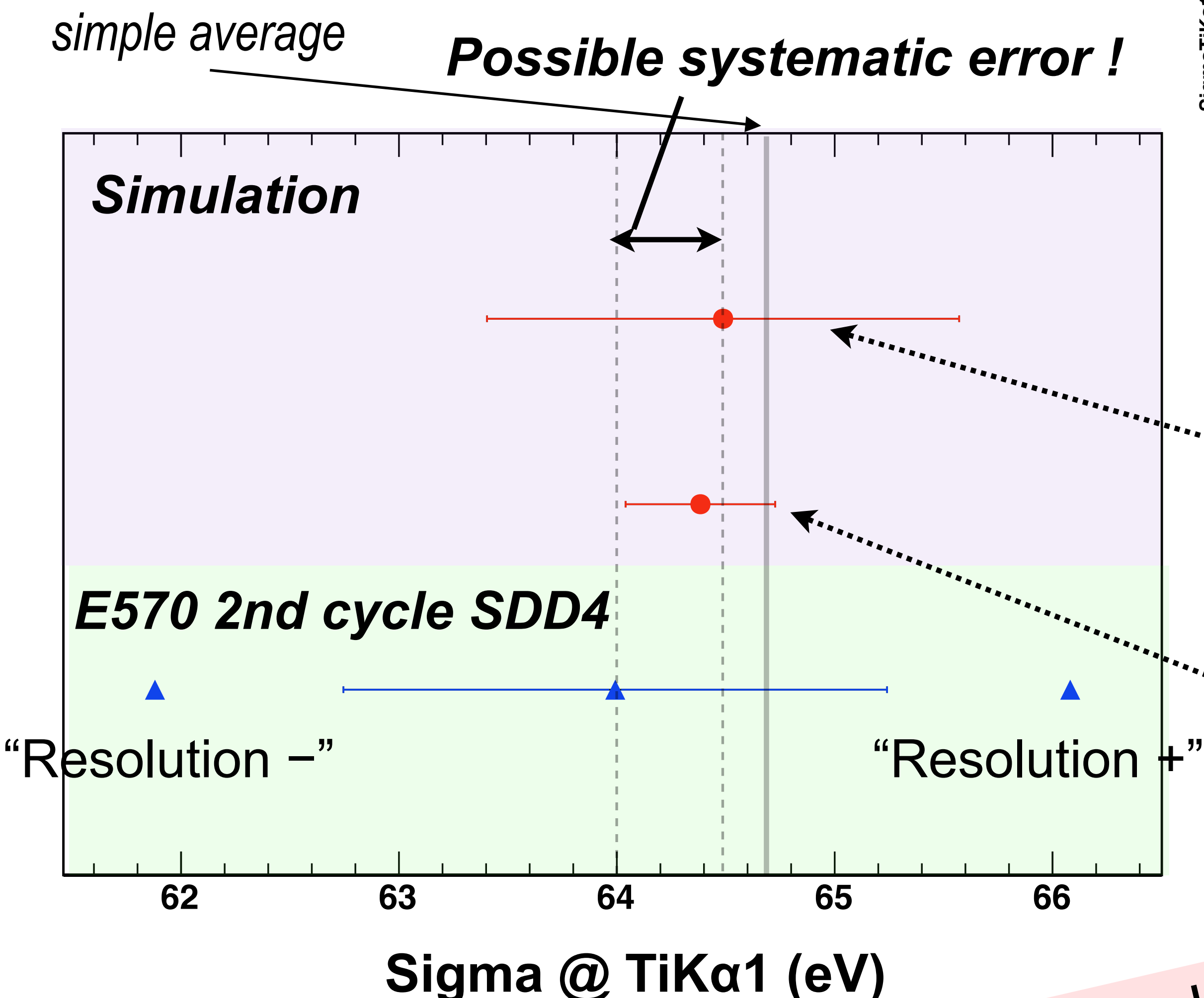
Expected value 2

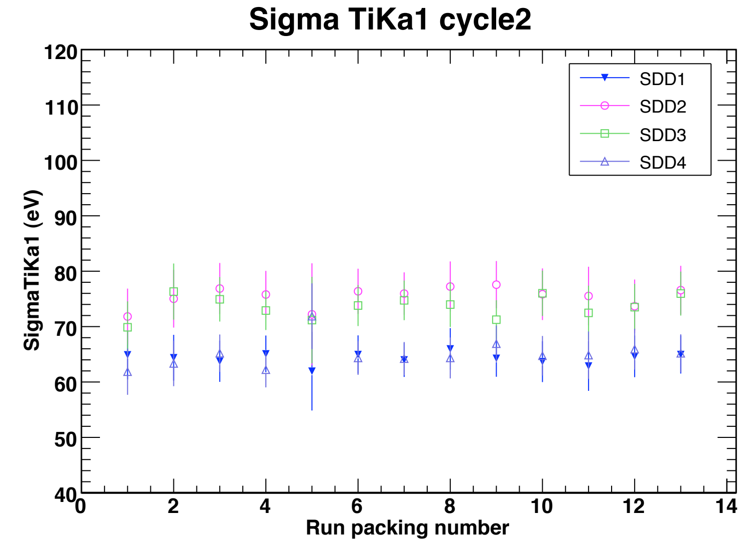
$$\mathrm{Gaus}(E, \sigma(t_i))$$

center value of σ

Good agreement with the simulation

2nd cycle SDD4 (TiKα1)





Expected value 1

$$\mathrm{Gaus}(E, \mathrm{Gaus}(\sigma(t_i), \sigma_i))$$

σ with its error

Expected value 2

$$\mathrm{Gaus}(E, \sigma(t_i))$$

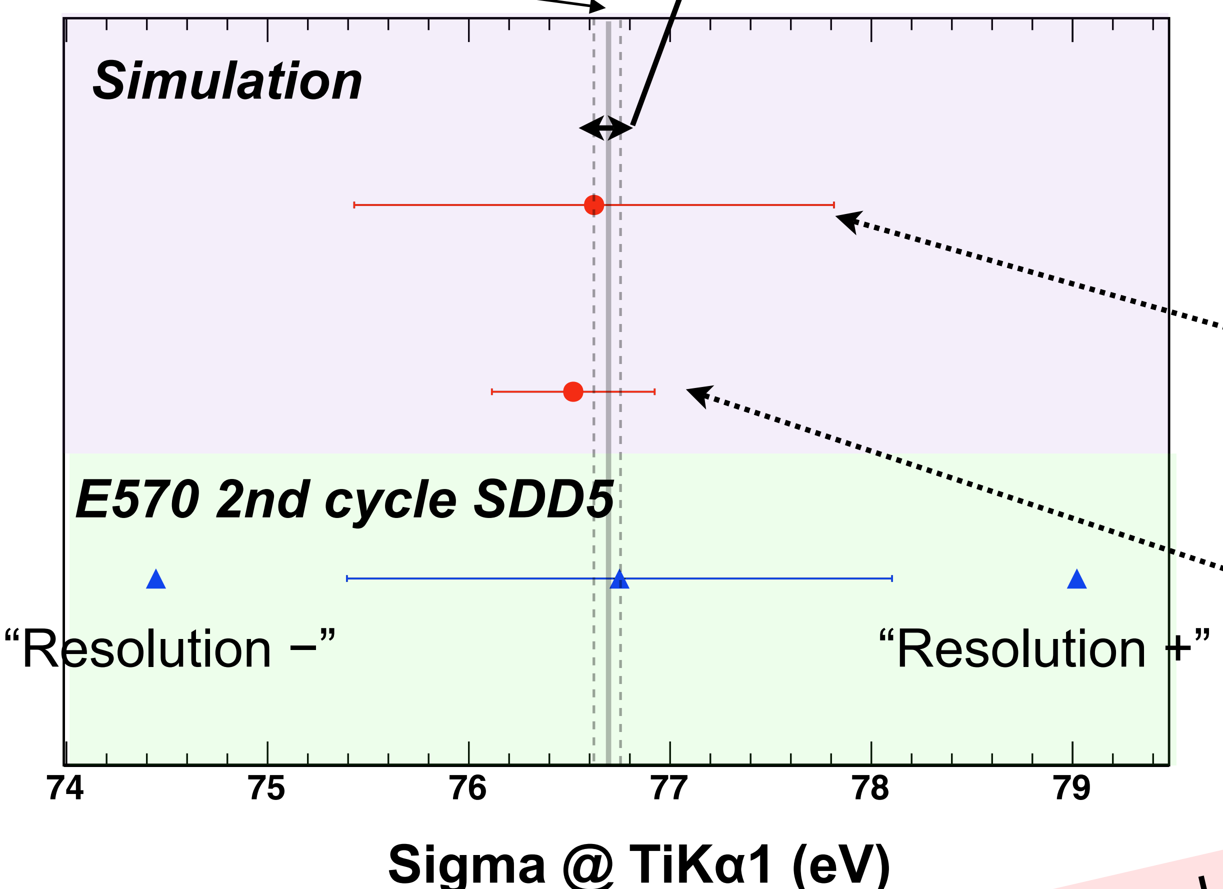
center value of σ

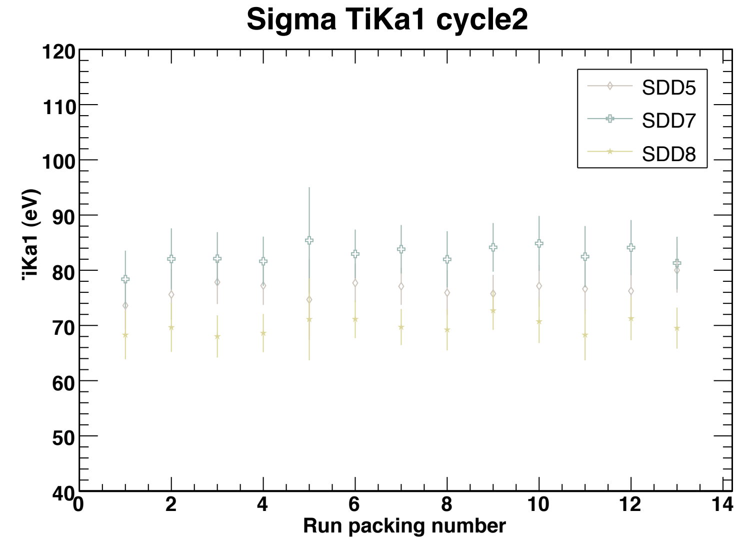
Good agreement with the simulation

2nd cycle SDD5 (TiKα1)

simple average

Possible systematic error !





Expected value 1

$$\mathrm{Gaus}(E, \mathrm{Gaus}(\sigma(t_i), \sigma_i))$$

σ with its error

Expected value 2

$$\mathrm{Gaus}(E, \sigma(t_i))$$

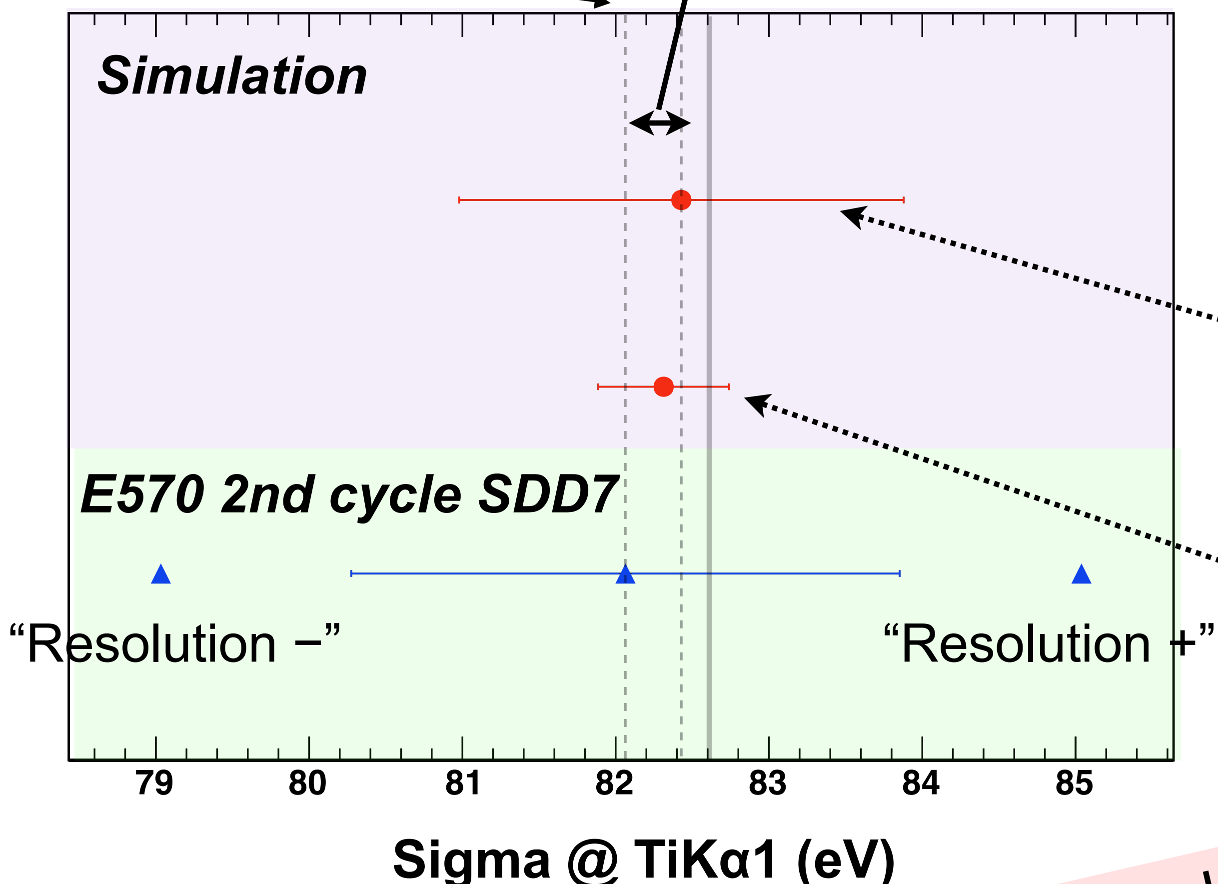
center value of σ

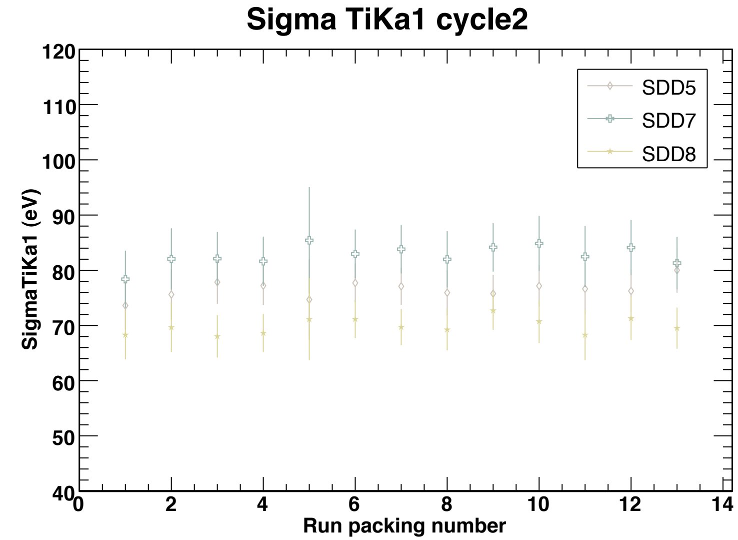
Good agreement with the simulation

2nd cycle SDD7 (TiKα1)

simple average

Possible systematic error !





Expected value 1

$$\mathrm{Gaus}(E, \mathrm{Gaus}(\sigma(t_i), \sigma_i))$$

σ with its error

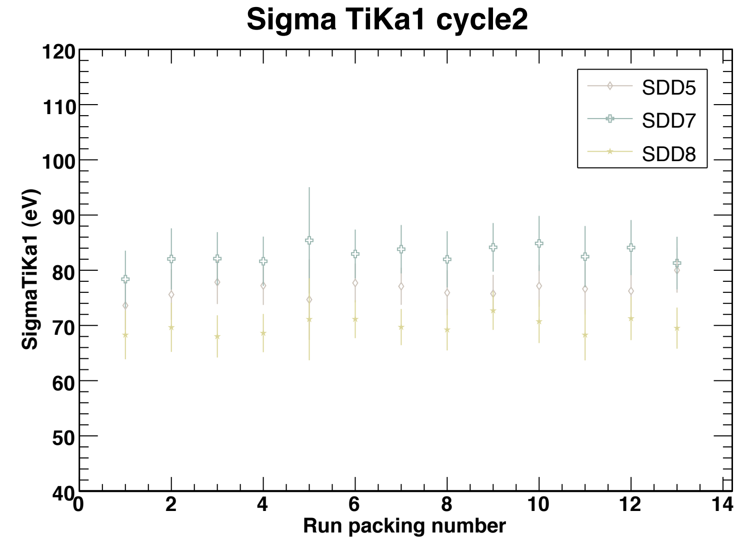
Expected value 2

$$\mathrm{Gaus}(E, \sigma(t_i))$$

center value of σ

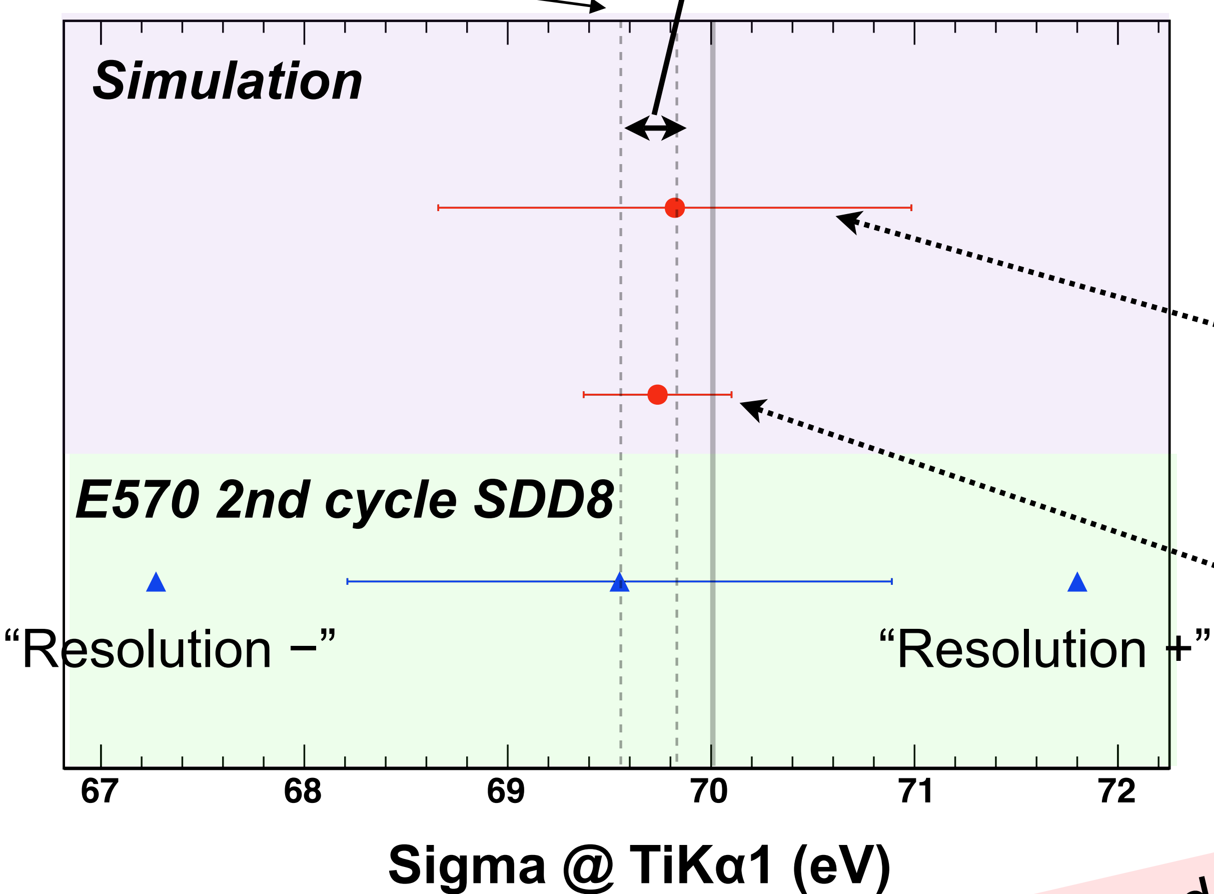
Good agreement with the simulation

2nd cycle SDD8 (TiKα1)



simple average

Possible systematic error !



Expected value 1

$$\mathrm{Gaus}(E, \mathrm{Gaus}(\sigma(t_i), \sigma_i))$$

σ with its error

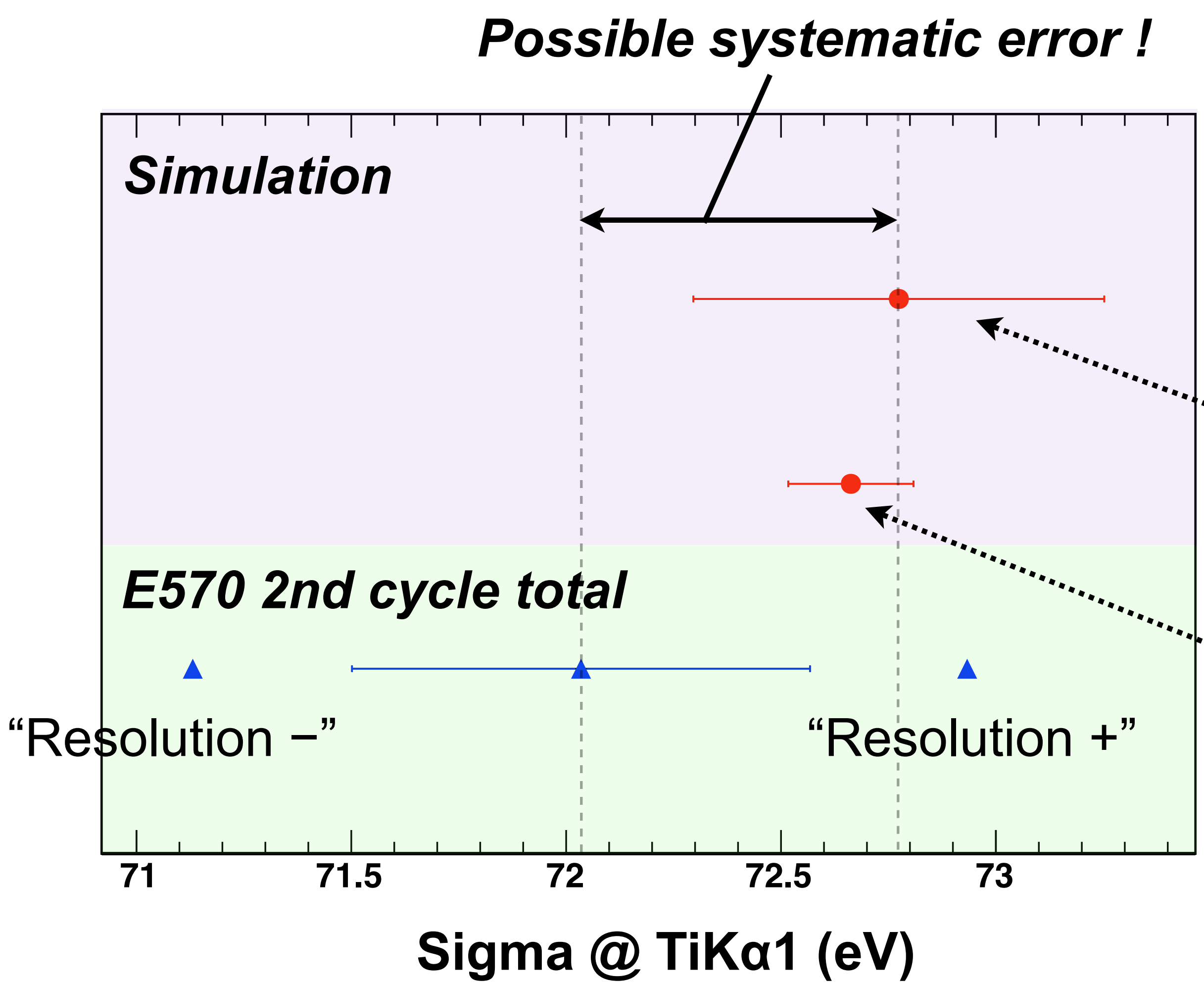
Expected value 2

$$\mathrm{Gaus}(E, \sigma(t_i))$$

center value of σ

Good agreement with the simulation

2nd cycle total (TiKα1)



Expected value 1

$\mathrm{Gaus}(E, \mathrm{Gaus}(\sigma(t_i), \sigma_i))$

σ with its error

Expected value 2

$\mathrm{Gaus}(E, \sigma(t_i))$

center value of σ