Formulation of normalization problems on the Λd spectra of KEK E549/570 Experiment

$\phi_{\Lambda d}$ - definition

Here, we try to give the concrete definition of $\phi_{\Lambda d}$, starting from the definition,

$$\phi_{\Lambda d}(p_\Lambda, p_d, \cos\theta_{\Lambda d}) = \frac{C_{\Lambda d}(p_\Lambda, p_d, \cos\theta_{\Lambda d})}{N_{stopK^-} \times \epsilon_{\Lambda d}(p_\Lambda, p_d, \cos\theta_{\Lambda d})}. \quad (1)$$

In order to consider directly available forms to the data, we do need to give the direct expressions of N_{stopK^-} and $\epsilon_{\Lambda d}$. One of them, N_{stopK^-} is generally given in the form,

$$N_{stopK^-} = \frac{N^-_{K\mu2}}{P^- \times Br \times \epsilon^-_\mu \epsilon^-_{DAQ} \epsilon^-_{BLC} \epsilon^-_{PDC} \epsilon^-_{TRG} \epsilon^-_{stopK} \epsilon^-_{delay}}, \quad (2)$$

where

$N^-_{K\mu2}$ **:** Observed $K\mu2$ peak number.

P^- **:** Free-decay-fraction of stopped K^- on ^{4}He.

Br **:** Branching ratio to the $K\mu2$ decay.

ϵ^-_μ **:** Overall detection efficiency of μ^- from $K\mu2$ decay.

ϵ^-_{DAQ} **:** DAQ live time rate for production runs.

ϵ^-_{BLC} **:** BLC tracking efficiency, for $K_{\mu2}$ decay event.

ϵ^-_{PDC} **:** PDC tracking efficiency, for $K_{\mu2}$ decay event.

ϵ^-_{TRG} **:** Efficiency of K^- identification by hardware trigger.

ϵ^-_{stopK} **:** Efficiency of stopped K^- identification by the analysis.

ϵ^-_{delay} **:** Kaon survival rate under the delayed time gate.

and concrete form of the $\epsilon_{\Lambda d}$ is considered case-by-case in the next section.

$\phi_{\Lambda d}$ - case-by-case expression

Here, we try to give a concrete form of $\epsilon_{\Lambda d}$. Now, let we define $\epsilon'_{\Lambda d}$, as the efficiency obtained neglecting the following factors, which is not taken into account in the acceptance calculation:

ϵ^{-}_{DAQ} **:** DAQ live time rate for production runs.

$\epsilon^{\Lambda d}_{BLC}$ **:** BLC tracking efficiency, for Λd events.

ϵ^{d}_{PDC} **:** PDC tracking efficiency for d, from Λd events.

ϵ^{p}_{PDC} **:** PDC tracking efficiency for p, from Λd events.

$\epsilon^{\pi^-}_{PDC}$ **:** PDC tracking efficiency for π^-, from Λd events.

ϵ^{p}_{VDC} **:** VDC tracking efficiency for p, from Λd events.

$\epsilon^{\pi^-}_{VDC}$ **:** VDC tracking efficiency for π^-, from Λd events.

ϵ^{-}_{TRG} **:** Efficiency of K^- identification by hardware trigger.

ϵ^{-}_{stopK} **:** Efficiency of stopped K^- identification by IDfunc>-1.

By using these factors, the relationship between $\epsilon_{\Lambda d}$ and $\epsilon'_{\Lambda d}$ is as follows:

$$case1 : \epsilon_{\Lambda d} = \epsilon^{-}_{DAQ} \cdot \epsilon^{\Lambda d}_{BLC} \cdot \epsilon^{d}_{PDC} \cdot \epsilon^{p}_{PDC} \cdot \epsilon^{\pi^-}_{VDC} \cdot \epsilon^{-}_{TRG} \cdot \epsilon^{-}_{stopK} \cdot \epsilon'_{\Lambda d}, \quad (3)$$

$$case2 : \epsilon_{\Lambda d} = \epsilon^{-}_{DAQ} \cdot \epsilon^{\Lambda d}_{BLC} \cdot \epsilon^{d}_{PDC} \cdot \epsilon^{\pi^-}_{PDC} \cdot \epsilon^{p}_{VDC} \cdot \epsilon^{-}_{TRG} \cdot \epsilon^{-}_{stopK} \cdot \epsilon'_{\Lambda d}, \quad (4)$$

$$missing\Lambda : \epsilon_{\Lambda d} = \epsilon^{-}_{DAQ} \cdot \epsilon^{\Lambda d}_{BLC} \cdot \epsilon^{d}_{PDC} \cdot \epsilon^{-}_{TRG} \cdot \epsilon^{-}_{stopK} \cdot \epsilon'_{\Lambda d}, \quad (5)$$

where **possible momentum and angular dependence of the tracking efficiencies had been neglected**. Therefore,

$$case1 : \frac{1}{N_{stopK^-} \cdot \epsilon_{\Lambda d}} = \frac{P^- \times Br \times \epsilon^{-}_{\mu}\epsilon^{-}_{DAQ}\epsilon^{-}_{BLC}\epsilon^{-}_{PDC}\epsilon^{-}_{TRG}\epsilon^{-}_{stopK}\epsilon^{-}_{delay}}{N^{-}_{K\mu 2}\epsilon^{-}_{DAQ}\epsilon^{\Lambda d}_{BLC}\epsilon^{d}_{PDC}\epsilon^{p}_{PDC}\epsilon^{\pi^-}_{VDC}\epsilon^{-}_{TRG}\epsilon^{-}_{stopK}\epsilon'_{\Lambda d}},$$

$$case2 : \frac{1}{N_{stopK^-} \cdot \epsilon_{\Lambda d}} = \frac{P^- \times Br \times \epsilon^{-}_{\mu}\epsilon^{-}_{DAQ}\epsilon^{-}_{BLC}\epsilon^{-}_{PDC}\epsilon^{-}_{TRG}\epsilon^{-}_{stopK}\epsilon^{-}_{delay}}{N^{-}_{K\mu 2}\epsilon^{-}_{DAQ}\epsilon^{\Lambda d}_{BLC}\epsilon^{d}_{PDC}\epsilon^{\pi^-}_{PDC}\epsilon^{p}_{VDC}\epsilon^{-}_{TRG}\epsilon^{-}_{stopK}\epsilon'_{\Lambda d}},$$

$$missing\Lambda : \frac{1}{N_{stopK^-} \cdot \epsilon_{\Lambda d}} = \frac{P^- \times Br \times \epsilon^{-}_{\mu}\epsilon^{-}_{DAQ}\epsilon^{-}_{BLC}\epsilon^{-}_{PDC}\epsilon^{-}_{TRG}\epsilon^{-}_{stopK}\epsilon^{-}_{delay}}{N^{-}_{K\mu 2}\epsilon^{-}_{DAQ}\epsilon^{\Lambda d}_{BLC}\epsilon^{d}_{PDC}\epsilon^{-}_{TRG}\epsilon^{-}_{stopK}\epsilon'_{\Lambda d}},$$

which are reduced into

$$
\begin{aligned}
case1 : \frac{1}{N_{stopK^-} \cdot \epsilon_{\Lambda d}} &= \frac{P^- \times Br \times \epsilon^-_{\mu} \epsilon^-_{BLC} \epsilon^-_{PDC} \epsilon^-_{delay}}{N^-_{K\mu 2} \epsilon^{\Lambda d}_{BLC} \epsilon^{d}_{PDC} \epsilon^{p}_{PDC} \epsilon^{\pi^-}_{VDC} \epsilon'_{\Lambda d}} \approx \frac{P^- \times Br \times \epsilon^-_{\mu} \epsilon^-_{delay}}{N^-_{K\mu 2} \epsilon^{d}_{PDC} \epsilon^{\pi^-}_{VDC} \epsilon'_{\Lambda d}}, \\
case2 : \frac{1}{N_{stopK^-} \cdot \epsilon_{\Lambda d}} &= \frac{P^- \times Br \times \epsilon^-_{\mu} \epsilon^-_{BLC} \epsilon^-_{PDC} \epsilon^-_{delay}}{N^-_{K\mu 2} \epsilon^{\Lambda d}_{BLC} \epsilon^{d}_{PDC} \epsilon^{\pi^-}_{PDC} \epsilon^{p}_{VDC} \epsilon'_{\Lambda d}} \approx \frac{P^- \times Br \times \epsilon^-_{\mu} \epsilon^-_{delay}}{N^-_{K\mu 2} \epsilon^{d}_{PDC} \epsilon^{p}_{VDC} \epsilon'_{\Lambda d}}, \\
missing\Lambda : \frac{1}{N_{stopK^-} \cdot \epsilon_{\Lambda d}} &= \frac{P^- \times Br \times \epsilon^-_{\mu} \epsilon^-_{BLC} \epsilon^-_{PDC} \epsilon^-_{delay}}{N^-_{K\mu 2} \epsilon^{\Lambda d}_{BLC} \epsilon^{d}_{PDC} \epsilon'_{\Lambda d}} \approx \frac{P^- \times Br \times \epsilon^-_{\mu} \epsilon^-_{delay}}{N^-_{K\mu 2} \epsilon'_{\Lambda d}},
\end{aligned}
$$

where we adopted approximations,

$$
\begin{aligned}
\epsilon^-_{BLC} &\approx \epsilon^{\Lambda d}_{BLC}, \\
\epsilon^-_{PDC} &\approx \epsilon^{\pi^-/p}_{PDC}.
\end{aligned}
$$

Introducing further approximation, $\epsilon^{d}_{PDC} \approx \epsilon^{p/\pi^-}_{VDC} \approx 1$, we obtain the final form,

$$
\phi_{\Lambda d}(p_\Lambda, p_d, \cos\theta_{\Lambda d}) = \frac{P^- \times Br \times \epsilon^-_{\mu} \times \epsilon^-_{delay}}{N^-_{K\mu 2}} \cdot \frac{C_{\Lambda d}(p_\Lambda, p_d, \cos\theta_{\Lambda d})}{\epsilon'_{\Lambda d}(p_\Lambda, p_d, \cos\theta_{\Lambda d})}. \tag{6}
$$

The normalization factor is found to be common for all cases if tracking efficiency for Λd events are approximated by 1.0. Hereafter, we adopt the expression commonly. Accordingly, expression of physical quantities and accompanying errors are reduced into

$$
\Phi_{\Lambda d}(f_0) = \frac{P^- \times Br \times \epsilon^-_{\mu} \times \epsilon^-_{delay}}{N^-_{K\mu 2}} \int\int\int_{f=f_0} dp_\Lambda dp_d d\cos\theta_{\Lambda d} \frac{C_{\Lambda d}(p_\Lambda, p_d, \cos\theta_{\Lambda d})}{\epsilon'_{\Lambda d}(p_\Lambda, p_d, \cos\theta_{\Lambda d})},
$$

$$
\Delta\Phi_{\Lambda d}(f_0) = \frac{P^- \times Br \times \epsilon^-_{\mu} \times \epsilon^-_{delay}}{N^-_{K\mu 2}} \int\int\int_{f=f_0} dp_\Lambda dp_d d\cos\theta_{\Lambda d} \frac{\sqrt{C_{\Lambda d}(p_\Lambda, p_d, \cos\theta_{\Lambda d})}}{\epsilon'_{\Lambda d}(p_\Lambda, p_d, \cos\theta_{\Lambda d})}.
$$