# An Estimation of the Acceptance of SDDs and the Count Rate of Calibration X-rays Using GEANT4 Simulation for the J-PARC DAY-1 Experiment 

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The acceptance of SDDs and the count rate of calibration X-rays are calculated using GEANT4 simulation for the proposal of the measurement of $K^{-}{ }^{3} \mathrm{He}$ atom X-rays at JPARC DAY-1. First, k-shell ionization cross section of Ti and Ni induced by $\pi^{-}$beam are calculated from the data of KEK-PS E570 experiment. Second, the $\pi^{-}$beam distribution near ${ }^{3} \mathrm{He}$ target on J-PARC K1.8BR beamline is simulated. Finally, the count rate of calibration X-rays are calculated. Assuming the $K / \pi$ ratio is 12.6 at $0.75 \mathrm{GeV} / c$ beam momentum, the obtained count rate is $\sim 200[/$ shift $/ \mathrm{SDD}]$ for Ti and Ni . This statistics is about $1 / 20$ of E 570 . To realize the comparable statistics of E 570 , the $K / \pi$ ratio must be less than 0.75 .

## 1 KEK-PS E570 experiment

The geometry of E570 near the liquid ${ }^{4} \mathrm{He}$ target is shown in Figure 1. In this experiment, Silicon Drift Detector (SDD) was used as an X-ray detector. SDD had $100 \mathrm{~mm}^{2}$ effective area and $260 \mu \mathrm{~m}$ thickness. 8 SDDs were installed symmetrically and tilted 45 degrees to see the center of the target and to reduce the $\pi^{-}$beam direct hit. Liquid ${ }^{4} \mathrm{He}$ target was cooled to the superfluid state (its density is $0.145 \mathrm{~g} / \mathrm{cm}^{3}$ ) and covered by $75 \mu \mathrm{~m}$ thin Mylar-film. The cylindrical target had 100 mm radius and 150 mm length. And to realize in-beam calibration, pure titanium and nickel foils were placed downstream of the target and around the target cell. The thickness of Ti foil was $50 \mu \mathrm{~m}$ and Ni one was $125 \mu \mathrm{~m}$. In
this simulation the aluminized Myler-film used as super-insulator is ignored.


Figure 1: KEK-PS E570 geometry near the liquid ${ }^{4} \mathrm{He}$ target.

## $1.1 \pi^{-}$beam profile

The $\pi^{-}$beam profile at $z=0 \mathrm{~mm}$ corresponding the center of target is shown in Figure 2. This profile is calculated from E549 data and the fitted Gaussian is used to generate the simulation beam.

The number of $\pi^{-}$which hits the foils is shown in Table 1 when 1 million $\pi^{-}$beam is generated. The X-rays are radiated when $\pi^{-}$hits the foils, and detected by SDD. The
total detector efficiency can be calculated from the number of the radiated X-rays and the detected X-rays.


Figure 2: The $\pi^{-}$beam profile at $z=0 \mathrm{~mm}$. This is realistic $\pi^{-}$beam profile of E549 experiment. The number of $\pi^{-}$is counted per spill ( 4.0 sec )

Table 1: The number of $\pi^{-}$which hits the foils, cone and ring, when 1 million $\pi^{-}$ beam is generated (this is a simulation result).

| foil | number of $\pi^{-}$[/spill] |
| :---: | :---: |
| cone Ti | 432754 |
| ring Ti | 18121 |
| cone Ni | 434062 |

### 1.2 Solid angle and efficiency of SDDs

The solid angle of SDDs are calculated by generating virtual particle "geantino" ${ }^{1}$ with uniform distribution The obtained acceptance is shown in the first column of Table 2. The blind corners of the target cell are considered.

The efficiency of SDDs can be calculated by generating the characteristic X-rays from corresponding foils and target considering the $\pi^{-}$beam and the stopped $K^{-}$distribution. All attenuation and absorption processes of X-rays are considered. The obtained efficiency is shown in the third column of Table 2. The density of the superfluid liquid ${ }^{4} \mathrm{He}$ target is $0.145 \mathrm{~g} / \mathrm{cm}^{3}$ so the transmission curve of X-rays on 90 mm length ${ }^{2}$ target becomes like Figure 3. Now the energy of the interesting X-ray is 6464 eV , so the averaged possibility of the transmission of the X-ray is about 0.7. For the calibration X-rays, the main component of the efficiency is their self attenuation.

Table 2: The solid angle, the possibility of transmission and the total efficiency of 7 SDDs (except for SDD6). For the target, $75 \mu \mathrm{~m}$ Mylar-film attenuation is included.

|  | acceptance [\%] | transmission [\%] | efficiency [\%] |
| :--- | :---: | :---: | :---: |
| cone Ti | 0.147 | 23.8 | 0.035 |
| ring Ti | 0.294 | 16.0 | 0.047 |
| cone Ni | 0.147 | 16.3 | 0.024 |
| target ${ }^{4} \mathrm{He}$ | 0.208 | 42.8 | 0.089 |

### 1.3 K-shell ionization cross section

From the efficiency of SDDs and the detected number of calibration X-rays, the k-shell ionization cross section can be calculated. The detected number of calibration X-rays are estimated by the E570 data summed up from run 529 to run 533 (nearly 1 shift $=8$ hours), and shown in Table 3.

[^0]

Figure 3: The transmission curve from 90 mm length superfluid liquid ${ }^{4} \mathrm{He}$. The energy of the interesting X-ray is 6464 eV . So the averaged possibility of the transmission is about 0.7 .

Table 3: The number of calibration X-rays ( $\mathrm{K} \alpha 1, \mathrm{~K} \alpha 2$, and $\mathrm{K} \beta$ ) estimated by E570 data summed up from run 529 to 533 and 7 SDDs (except for SDD6).

| X-rays | detected number [/shift] | $\rightarrow[/$ shift/SDD] | $\rightarrow[/$ spill/SDD] |
| :--- | :---: | :---: | :---: |
| Ti | 31551 | 4507 | 0.626 |
| Ni | 33442 | 4777 | 0.663 |

The k -shell ionization can be calculated by the following equation,

$$
\begin{equation*}
\sigma_{K}=N_{o b s} /\left(N_{\pi} \cdot N_{A} \frac{d}{A} t \cdot \epsilon \cdot \omega_{K}\right) \tag{1}
\end{equation*}
$$

where $N_{o b s}$ is the detected number of X-rays, $N_{\pi}$ is the number of $\pi^{-}$which hits the foil, $N_{A}$ is Avogadro's constant, $d$ is the density of the foil material, $A$ is the atomic weight, $t$ is the foil thickness, $\epsilon$ is the detector efficiency and $\omega_{K}$ is the fluorescence yield. For Ti and Ni constants are listed up in Table 4.

Table 4: The constants of Ti and Ni.

| material | $d\left[\mathrm{~g} / \mathrm{cm}^{3}\right]$ | $A[\mathrm{~g} / \mathrm{mol}]$ | $t[\mu \mathrm{~m}]$ | $\omega_{K}$ |
| :---: | :---: | :---: | :---: | :---: |
| Ti | 4.54 | 47.867 | 50 | 0.219 |
| Ni | 8.91 | 58.70 | 125 | 0.414 |

The number of $\pi^{-}$which hits the foil is already simulated, and the detected number of X-rays is also calculated. Considering TKO acceptance as $\sim 0.73, N_{\text {obs }}$ must be divided by 0.73. Then the k -shell ionization cross section is

$$
\begin{align*}
\sigma_{K}(\mathrm{Ti})= & 0.626 \times 7 \div\left(432754 \times \frac{0.035}{100} \times \frac{0.005}{\sin \theta}+18121 \times \frac{0.047}{100} \times 0.005\right) \\
& \div\left(6.02 \times 10^{23} \times \frac{4.54}{47.867} \times 0.219 \times 0.73\right) \\
= & 505[\text { barn }] \tag{2}
\end{align*}
$$

where $\theta=\arctan (73.0 / 48.3)$ is the foil angle along $z$ axis. In the same way,

$$
\begin{align*}
\sigma_{K}(\mathrm{Ni})= & 0.663 \times 7 \div\left(434062 \times \frac{0.024}{100} \times 0.0125\right) \\
& \div\left(6.02 \times 10^{23} \times \frac{8.91}{58.70} \times 0.414 \times 0.73\right) \\
= & 129[\text { barn }] \tag{3}
\end{align*}
$$

Compared with PIXE data measured by 1 GeV proton beam (Figure 4), the present cross section is mostly consistent. Now the dead time of SDD is ignored in the detected number of X-rays. The obtained cross section can increase a few percent by considering the dead time of SDDs.


Figure 4: K-shell ionization cross sections for 1 GeV proton projectiles[1]. The full curve is derived from the calculations based on the formalism by Anholt (1979). The chain curve is the longitudinal Coulomb contribution in PWBA theory.

## 2 J-PARC DAY-1 experiment

### 2.1 Geometry

The geometry of J-PARC DAY-1 experiment near the liquid ${ }^{3} \mathrm{He}$ target is shown in Figure 5. In this experiment, $200 \mu \mathrm{~m}$ beryllium window is used as target cell. The radius of the liquid ${ }^{3} \mathrm{He}$ target is 32 mm , the length is 150 mm and the density is $0.08 \mathrm{~g} / \mathrm{cm}^{3}$. This time SDDs are installed on the side of the target, the distance between the center of target and the surface of SDD is 100 mm . The calibration foils are placed around the target cell as doughnut shape. The thickness of Ti foil is $50 \mu \mathrm{~m}$ and Ni one is $125 \mu \mathrm{~m}$.


Figure 5: J-PARC DAY-1 experiment geometry near the liquid ${ }^{3} \mathrm{He}$ target.

## $2.2 \pi^{-}$beam profile

The $\pi^{-}$beam profile at $z=-55 \mathrm{~mm}$ (Ti foil) and $z=-45 \mathrm{~mm}$ (Ni foil) is shown in Figure 6 and Figure 7 respectively. This profile was calculated from TURTLE and GEANT4. The $\pi^{-}$beam distribution and direction at the final focus point are simulated using TURTLE, and the beam profile at the place of calibration foils is reconstructed using GEANT4 assuming the beam momentum is $0.75 \mathrm{GeV} / \mathrm{c}$ and the momentum bite is $\pm 2.5 \%$.

The number of $\pi^{-}$which hits the foils when 1 million beam is generated is shown in Table 5. And the per spill ${ }^{3}$ hit rate is calculated assuming the intensity of K 1.8 BR beamline is $0.19 \mathrm{M} /$ spill.

In the same way of E570, the solid angle and the efficiency can be calculated from the hit number of $\pi^{-}$and the detected number of X-rays.


Figure 6: The $\pi^{-}$beam profile at $z=-55 \mathrm{~mm}$ (Ti foil is placed here) simulated by TURTLE and GEANT4. The number of counts is not realistic.

[^1]

Figure 7: The $\pi^{-}$beam profile at $z=45 \mathrm{~mm}$ (Ni foil is placed here) simulated by TURTLE and GEANT4. The number of counts is not realistic.

Table 5: The number of $\pi^{-}$which hits the foils when 1 million beam is generated (this is simulation result). In the calculation of per spill hit rate, assuming the intensity of K 1.8 BR beamline is $0.19 \mathrm{M} /$ spill and the $K / \pi$ ratio is 12.6 .

| foil | number of $\pi^{-}$ | $\rightarrow[/$ spill $]$ |
| :--- | :---: | :---: |
| $\mathrm{Ti} z=-55 \mathrm{~mm}$ | 694808 | 10477 |
| $\mathrm{Ni} z=45 \mathrm{~mm}$ | 702839 | 10598 |

### 2.3 Solid angle and efficiency of SDDs

The solid angle and the efficiency of SDDs is shown in Table 6. The blind corners of the target cell are considered.

The efficiency of SDDs can be calculated by generating the characteristic X-rays from corresponding foils considering the $\pi^{-}$beam distribution. In the estimation of target, the stopped $K^{-}$distribution is assumed as uniform. All attenuation and absorption processes of X-rays are considered.

Table 6: The solid angle, the possibility of transmission and the total efficiency of 8 SDDs. For the target the attenuation of beryllium window is included.

|  | acceptance [\%] | transmission [\%] | efficiency [\%] |
| :--- | :---: | :---: | :---: |
| $\mathrm{Ti} z=-55$ | 0.314 | 25.5 | 0.080 |
| $\mathrm{Ni} z=45$ | 0.386 | 10.9 | 0.042 |
| target $^{3} \mathrm{He}$ | 0.473 | 90.3 | 0.427 |

### 2.4 Count rate of calibration X-rays

The k-shell ionization cross sections of Ti and Ni are calculated from E570 experiment data. So the count rate of calibration X-rays can be obtained. The equation is,

$$
\begin{equation*}
N_{o b s}=N_{\pi} \sigma_{K} \omega_{K} N_{A} \frac{d}{A} t \epsilon \tag{4}
\end{equation*}
$$

where $N_{\text {obs }}$ is the detected number of X-rays, $N_{\pi}$ is the number of $\pi^{-}$which hits the foil, $N_{A}$ is Avogadro's constant, $d$ is the density of the foil material, $A$ is the atomic weight, $t$ is the foil thickness, $\epsilon$ is the detector efficiency and $\omega_{K}$ is the fluorescence yield. Now assuming the TKO acceptance is $100 \%$. Then for Ti ,

$$
\begin{align*}
N_{\text {obs }}(\mathrm{Ti}) & =10477 \times 500 \times 0.219 \times 6.02 \times 10^{23} \times \frac{4.54}{47.867} \times 0.005 \times 0.080 / 100 \\
& =0.26[/ \text { spill }] \\
& =267[/ \text { shift } / \mathrm{SDD}] \tag{5}
\end{align*}
$$

In the same way,

$$
\begin{align*}
N_{\text {obs }}(\mathrm{Ni}) & =10598 \times 100 \times 0.414 \times 6.02 \times 10^{23} \times \frac{8.91}{58.70} \times 0.0125 \times 0.042 / 100 \\
& =0.21[/ \text { spill }] \\
& =214[/ \text { shift } / \mathrm{SDD}] \tag{6}
\end{align*}
$$

these count rate are $\sim 1 / 20$ of the count rate of E 570 . To realize the comparable statistics of E570, the $K / \pi$ ratio must be less than 0.75 .

## References

[1] I B Vodopyanov, V V Pashuk and M V Stabnikov J. Phys. B: At. Mol. Opt. Phys. 29 (1996) 2543-2552


[^0]:    1 "geantino" is a virtual particle of GEANT4. It has no interaction and no mass.
    ${ }^{2}$ This length is the averaged distance between the center of target and SDD.

[^1]:    ${ }^{3}$ The spill of J-PARC is defined as 3.53 sec .

