

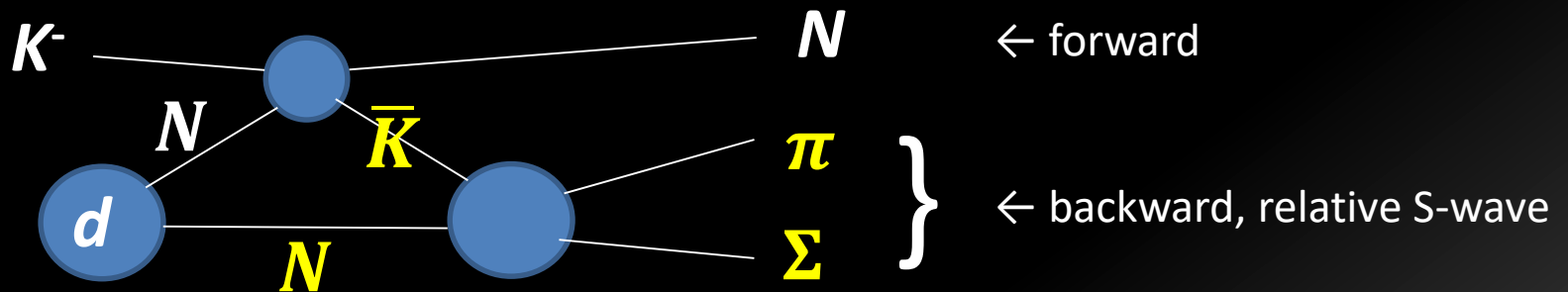
$d(K^-, n)\pi\Sigma_{I=0}$ スペクトルの解析

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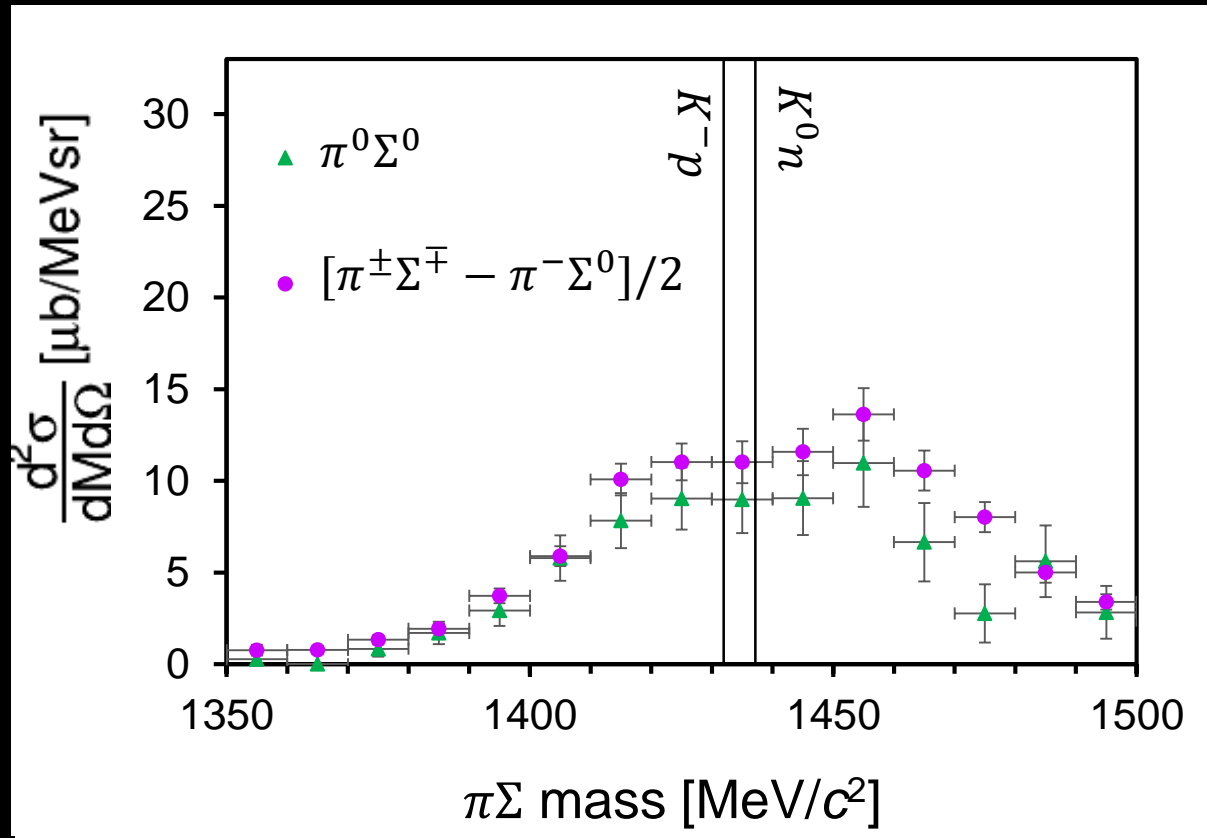
$K^{\text{bar}}N$ scattering below the $K^{\text{bar}}N$ thres. (J-PARC E31)



- measuring an **S-wave $\bar{K}N \rightarrow \pi\Sigma$** scattering below the $\bar{K}N$ threshold in the $d(K^-, n)\pi\Sigma$ reactions at a forward angle of N .
- ID's all the final states to decompose the $l=0$ and 1 ampl's.

	Fwd N	$\pi\Sigma$ mode	Isospin	Expected resonance
井上 {	n	$\pi^\pm \Sigma^\mp$	$0, 1$	$\Lambda(1405)$ interference btw $l=0$ and 1 ampl's.
	p	$\pi^- \Sigma^0$	1	P-wave $\Sigma^*(1385)$ to be suppressed
川崎	n	$\pi^0 \Sigma^0$	0	$\Lambda(1405)$

$$[\pi^{\pm}\Sigma^{\mp} - \pi^{-}\Sigma^0]/2 \text{ vs } \pi^0\Sigma^0 (I' = 0)$$

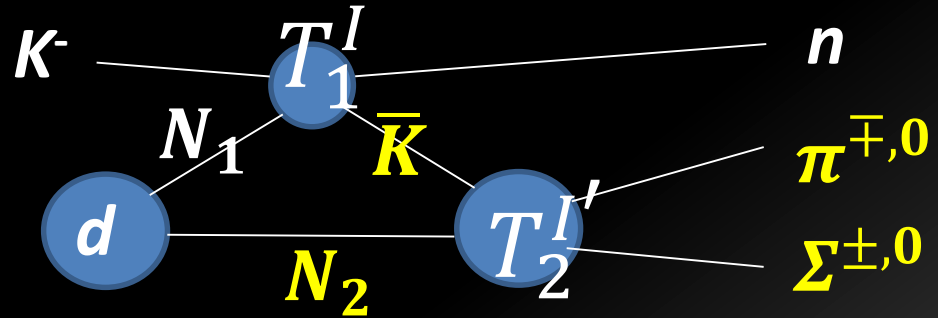


$$\frac{d\sigma}{d\Omega}([\pi^{\pm}\Sigma^{\mp} - \pi^{-}\Sigma^0]/2) \propto \left| -\frac{3T_1^{I=0} - T_1^{I=1}}{4\sqrt{3}} T_2^{I'=0} \right|^2 \approx \frac{d\sigma}{d\Omega}(\pi^0\Sigma^0) \propto \left| -\frac{3T_1^{I=0} - T_1^{I=1}}{4\sqrt{3}} T_2^{I'=0} \right|^2$$

Isospin relation seems to be satisfied.

Extracting Scattering Amplitude

- 2-step process



$$\frac{d\sigma}{dM_{\pi\Sigma}} \Big|_{\theta_n=3^\circ} \sim \left| \left\langle n\pi\Sigma \left| T_2^{I'} (\bar{K}N_2 \rightarrow \pi\Sigma) G_0 T_1^I (K^-N_1 \rightarrow \bar{K}n) \right| K^- \Phi_d \right\rangle \right|^2$$

$$\sim \left| T_2^{I'} (\bar{K}N \rightarrow \pi\Sigma) \right|^2 F_{\text{res}}(M_{\pi\Sigma})$$

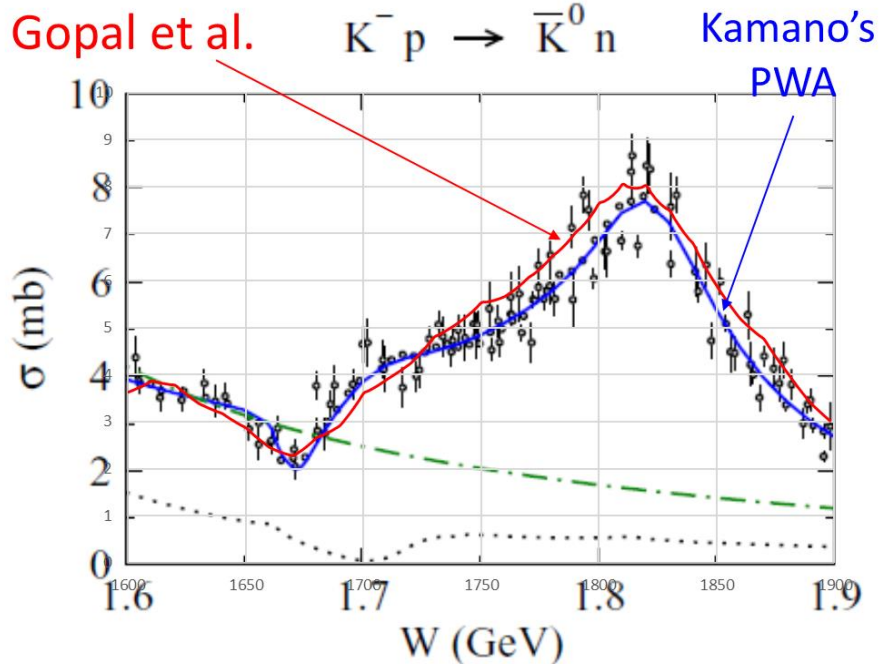
Factorization Approximation

$$F_{\text{res}}(M_{\pi\Sigma}) \sim \left| \int_0^\infty dq_{N_2}^3 T_1^I \frac{1}{E_{\bar{K}} - E_{\bar{K}}(q_{\bar{K}}) + i\epsilon} \Phi_d(q_{N_2}) \right|^2, q_{\bar{K}} + q_{N_2} = q_{\pi\Sigma}$$

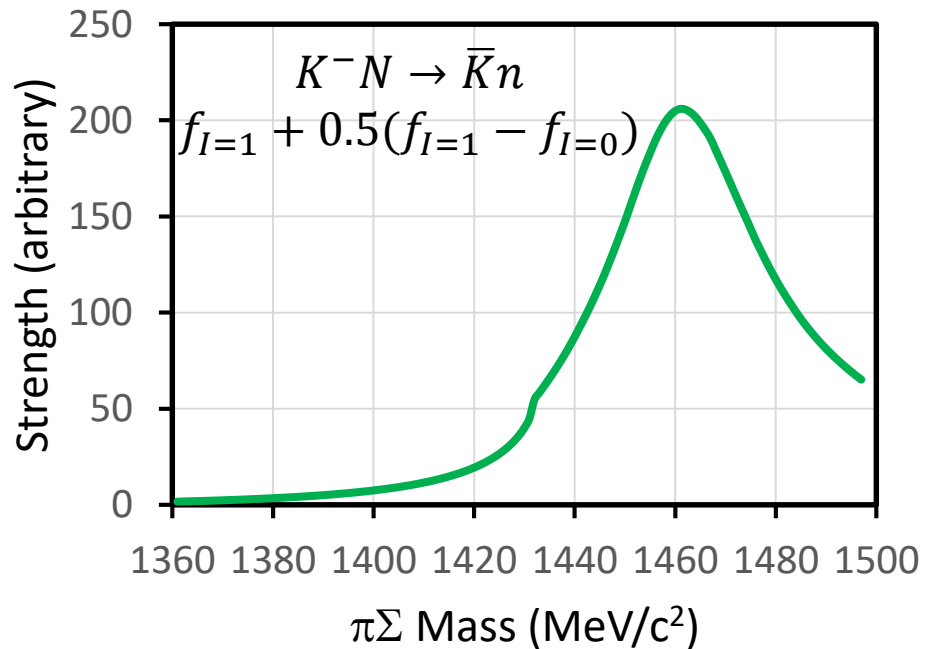
E31: Response Function, $F_{\text{res}}(M_{\pi\Sigma})$

$$F_{\text{res}}(M_{\pi\Sigma}) \sim p_{\pi}^{cm} p_n^2 / |(E_{K^-} + m_d)\beta_n - p_{K^-} \cos \theta| \times \left| \int d\Omega_{\pi}^{cm} E_{\pi} E_{\Sigma} \left| \int q_2 T_1^I(p_{K^-}, q_N, p_n, q_{\bar{K}}, \cos \theta_{n\bar{K}}; M_{\pi\Sigma}) G_0(q_2, q_1) \Phi_d(q_2) d^3 q_2 \right|^2 \right.$$

Elementary Cross Section for T_1^I



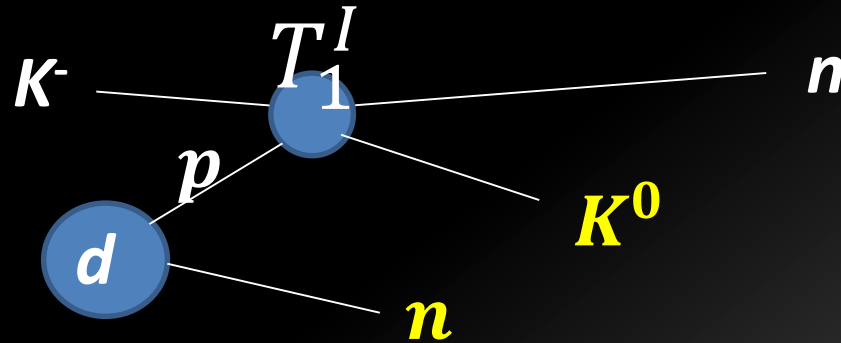
$F_{\text{res}}(M_{\pi\Sigma})$



Gopal et al., NPB119, 362(1977)

Demonstration of the T_1^I amplitude

- 1-step process

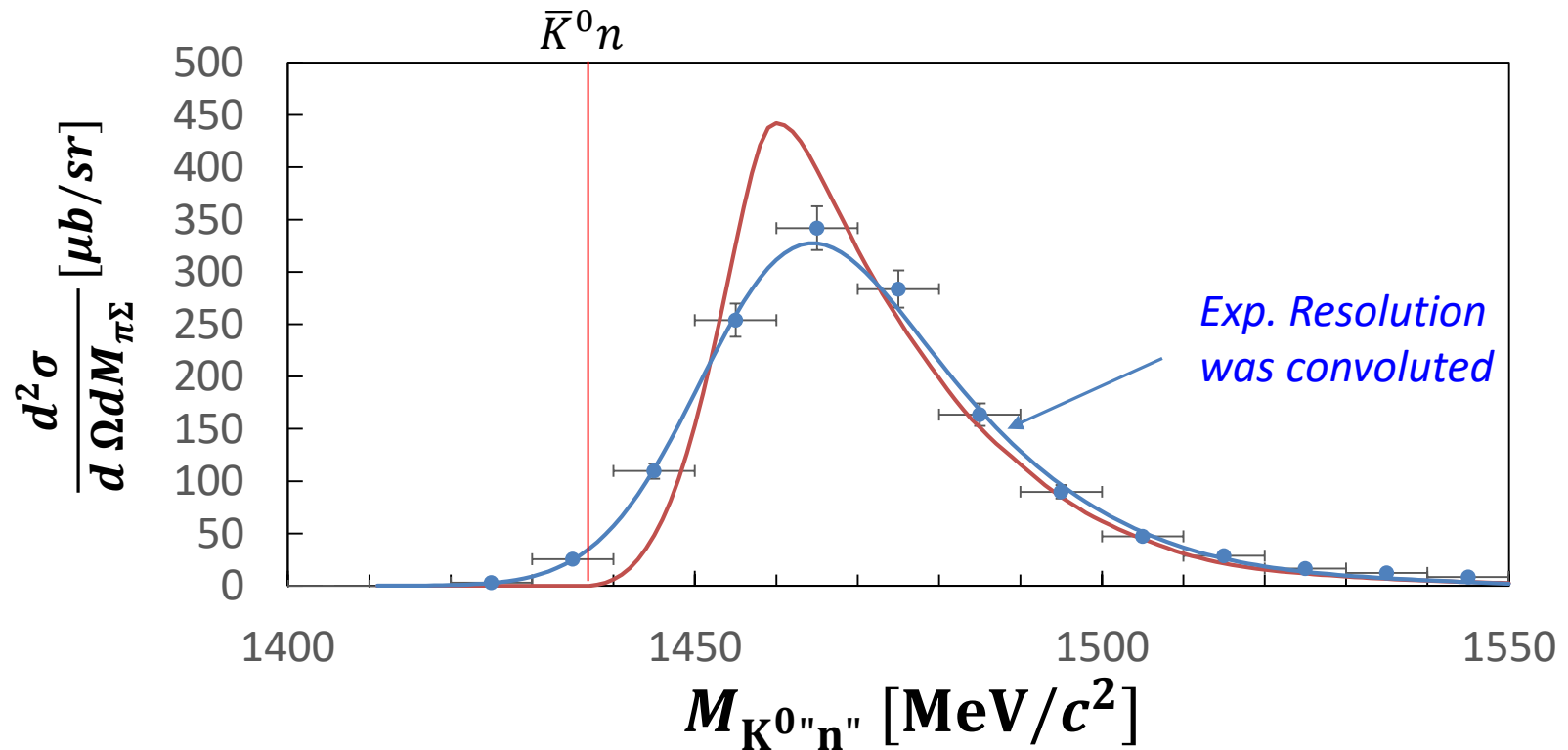


$$\frac{d\sigma}{dM_{\pi\Sigma}} \Big|_{\theta_n=3^\circ} \sim |\langle nK^0 n | T_1^I (K^- p \rightarrow \overline{K^0} n) | K^- \Phi_d \rangle|^2$$

$$\frac{d\sigma}{dM_{\pi\Sigma}} \sim \left| \int_0^\infty dq_{N_2}^3 T_1^I \delta(p_{K^-} + p_p - p_n - p_{K^0}) \Phi_d(q_{N_2}) \right|^2$$

Demonstration for fitting data with the 1-step $K^- d \rightarrow n K^0 n$ reaction calculation

- Data: $d(K^-, n) \bar{K}^0 n$ Ks/KL, BR(Ks- \rightarrow pi+-) corrected (K. Inoue)



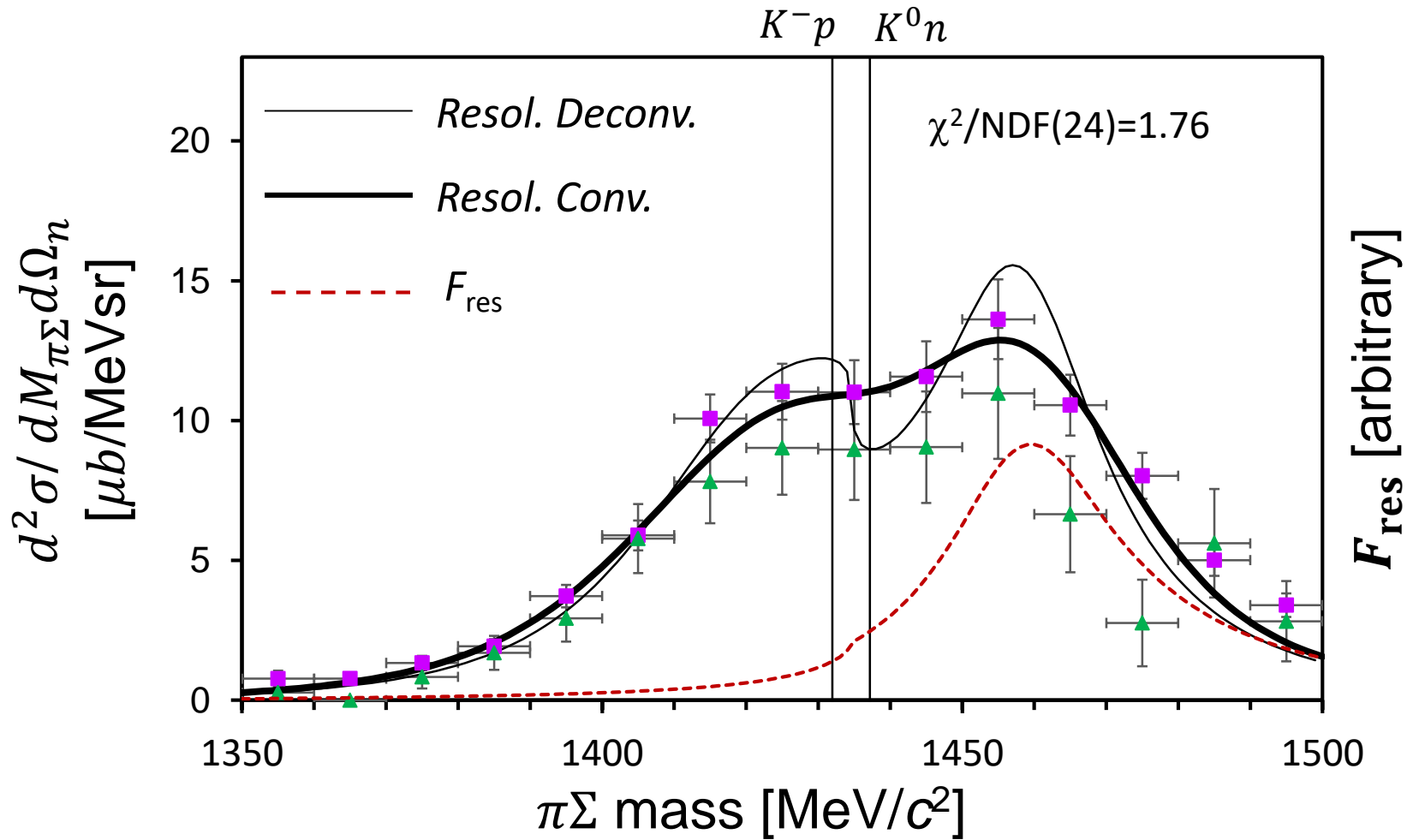
$\bar{K}N$ Scattering Amplitude

L. Lensniak, arXiv:0804.3479v1(2008)

- $T_2^I(\bar{K}N \rightarrow \bar{K}N) = \frac{A}{1 - iAk_2 + \frac{1}{2}ARk_2^2}$
- $T_2^I(\bar{K}N \rightarrow \pi\Sigma) = \frac{1}{\sqrt{k_1}} e^{i\delta_0} \frac{\sqrt{\text{Im}A - \frac{1}{2}|A|^2 \text{Im}Rk_2^2}}{1 - iAk_2 + \frac{1}{2}ARk_2^2}$
- $T_2^I(\pi\Sigma \rightarrow \pi\Sigma)$

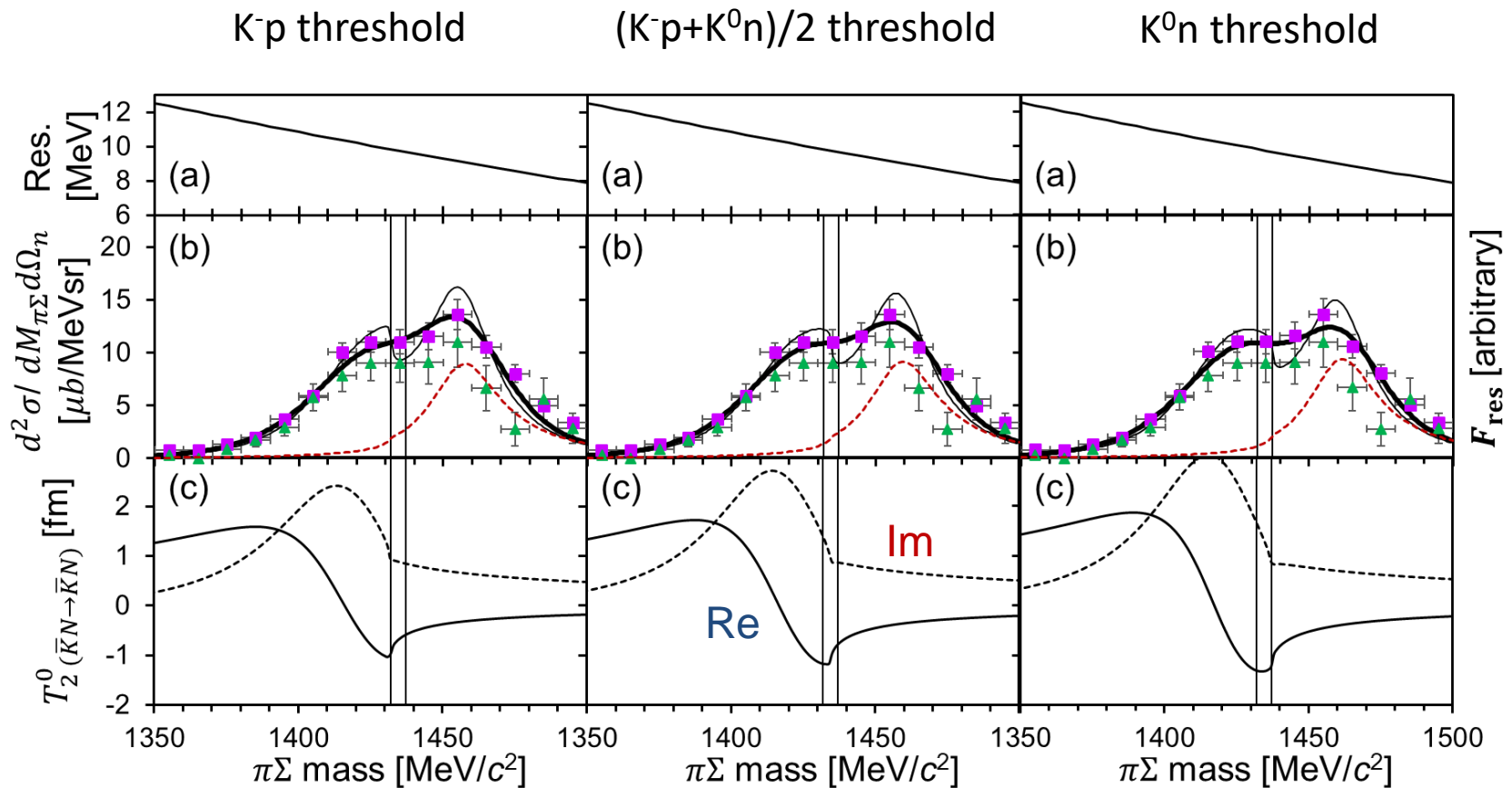
$$= \frac{e^{i\delta_0}}{k_1} \frac{\left(\sin \delta_0 + i \text{Im}(e^{-i\delta_0} A)k_2 - \frac{1}{2} \text{Im}(e^{-i\delta_0} AR)k_2^2 \right)}{1 - iAk_2 + \frac{1}{2}ARk_2^2}$$
- 5 real number parameters (effective range expansion)
 - A : scattering length, R : effective range, δ_0 : phase

Fit the spectra to deduce $\bar{K}N$ scattering amplitude



Systematics of the fitting result by the assumed $\bar{K}N$ mass threshold

$$\left. \frac{d\sigma}{dM_{\pi\Sigma}} \right|_{\theta_n=0} \sim \left| T_2^{I'}(\bar{K}N \rightarrow \pi\Sigma) \right|^2 F_{\text{res}}(M_{\pi\Sigma})$$



Best fit $\bar{K}N$ scattering amplitude

Scattering Length $A^{I=0} = (-1.12 \pm 0.11_{-0.07}^{+0.10}) + i(0.84 \pm 0.12_{-0.07}^{+0.08})$ fm

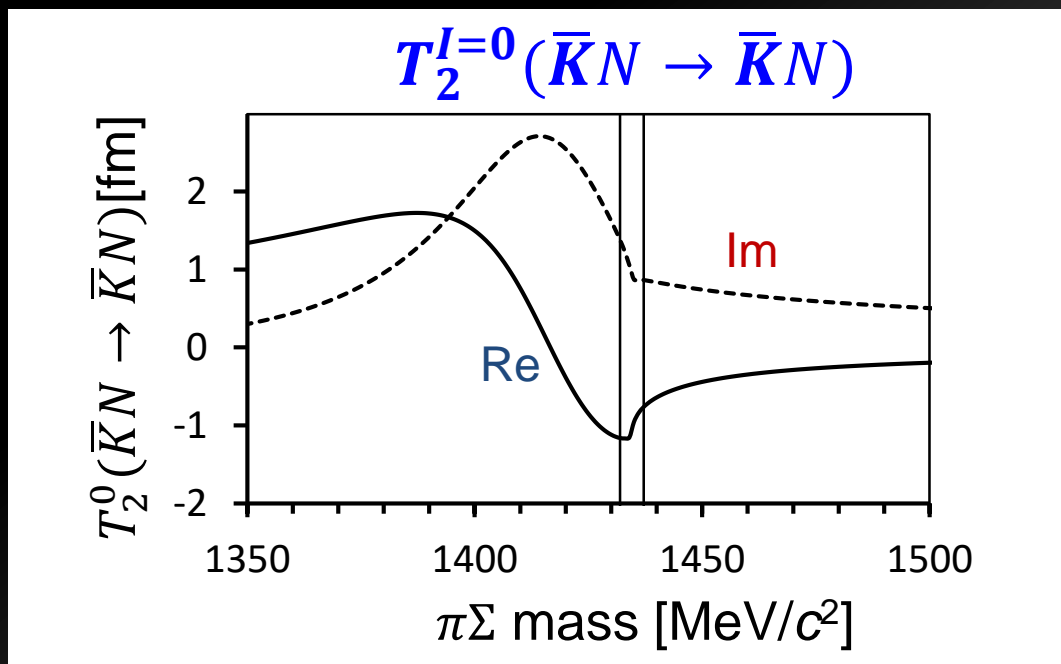
Effective Range $R^{I=0} = (-0.18 \pm 0.31_{-0.06}^{+0.08}) + i(0.41 \pm 0.13_{-0.09}^{+0.09})$ fm

A pole at $(1417.7_{-7.4-1.0}^{+6.0+1.1}) + (-26.1_{-7.9-2.0}^{+6.0+1.7})i$ MeV/c²

$$|T_2^{I=0}(\bar{K}N \rightarrow \bar{K}N)|^2 / |T_2^{I=0}(\bar{K}N \rightarrow \pi\Sigma)|^2 = 2.2_{-0.6-0.3}^{+1.0+0.3}$$

*best fit value \pm fitting error \pm systematic error

systematic errors assuming the K^-p/K^0n mass threshold



Conclusion

- We analyzed the $I = 0$ $\pi\Sigma$ mass spectra in the $K^-d \rightarrow N\pi\Sigma$ reactions, knocked-out N measured at ~ 0 degree.
 - well described with the two-step reaction process,
 $K^-N_1 \rightarrow N\bar{K}, \bar{K}N_2 \rightarrow \pi\Sigma$
 - S -wave $\bar{K}N_2 \rightarrow \pi\Sigma$ scattering is dominant.
- S -wave $\bar{K}N$ scattering amplitude in $I = 0$ was deduced.
 - found a resonance pole at $1417.7 - 26.1i$ [MeV], which is likely to couple to the $\bar{K}N$ state.

What's next:

Spectral analysis in a wider kinematical region by H. Asano

Backup

Comparison w/ Recent Work

	$A^{I=0}$ [fm]	Pole 1 [MeV]	Pole2 [MeV]	reference
This work	$-1.12 \pm 0.11_{-0.07}^{+0.10}$ $+i0.84 \pm 0.12_{-0.07}^{+0.08}$	$1417.7_{-7.4-1.0}^{+6.0+1.1}$ $-i26.1_{-6.0-1.7}^{+7.9+2.0}i$		
IHW	$-1.97 + 1.05i$ $^{\$)}$	$1424_{-23}^{+7} - i26_{-14}^{+3}$	$1381_{-6}^{+18} - i81_{-8}^{+19}$	ChPT full NLO, $\bar{K}N$ scatt., SIDDHARTA data constraint
TW1 (NLO30)	$-1.61 + 1.02i$ $^{\$)}$	$1433 - i25$ ($1418 - i44$)	$1371 - i54$ ($1355 - i86$)	ChPT LO(NLO), $\bar{K}N$ scatt., SIDDHARTA data constraint
MM#4 (#2)	$-1.81_{-0.28}^{-0.30}$ $+i0.92_{-0.23}^{+0.29}$ $^{\#)}$	$1429_{-7}^{+8} - i12_{-3}^{+2}$ ($1434_{-2}^{+2} - i10_{-1}^{+2}$)	$1325_{-15}^{+15} - i90_{-18}^{+12}$ ($1330_{-5}^{+4} - i56_{-11}^{+17}$)	ChPT NLO, $\bar{K}N$, CLAS and SIDDHARTA data constraint
GO Fit-II	$-1.79_{-0.14}^{+0.13}$ $+i1.36_{-0.19}^{+0.18}$	$1421_{-2}^{+3} - i19_{-5}^{+8}$	$1388_{-9}^{+9} - i114_{-25}^{+24}$	ChPT, $\bar{K}N$ scatt., SIDDHARTA data constraint
\sqrt{E} -dep	$-1.89 + 1.11i$ $^{\$)}$	$1429 - i15$	$1344 - i49$	ChPT LO, $\bar{K}N$ scatt., SIDDHARTA data constraint
LQCD	$-1.77 + 1.08i$	$1430 - i21$	$1338 - i89$	Lattice QCD

IHW: Ikeda, Hyodo, Weise, NPA **881**(2012)98, $^{\$)}$ value found in PRC **97**(2019)055209

TW1: Cieply, Smejkal, NPA **881**(2012)115, $^{\$)}$ value found in PRC **97**(2019)055209

GO Fit-II: Guo, Oller, PRC **87**(2013)035202

MM#4: Mai, Meissner, EPJA **51**(2015)30, $^{\#)}$ NPA900(2013)51

\sqrt{E} -dep: Ohnishi, Ikeda, Hyodo, Weise, PRC **93**(2016)025207, $^{\$)}$ value found in PRC **97**(2019)055209

LQCD: Liu, Wu, Leinweber, Thomas, PLB **808**(2020)135652

Liu, Hall, Leinweber, Thomas, Wu, PRD **95**(2017)014506

Pole Structure of the Lambda(1405) Region

PDG Reviews: Ulf-G. Meissner and T. Hyodo (since Nov. 2015)

Table 83.1: Comparison of the pole positions of $\Lambda(1405)$ in the complex energy plane from next-to-leading order chiral unitary coupled-channel approaches including the SIDDHARTA constraint. The lower two results also include the CLAS photoproduction data.

approach	pole 1 [MeV]	pole 2 [MeV]
Refs. [14, 15], NLO	$1424_{-23}^{+7} - i 26_{-14}^{+3}$	$1381_{-6}^{+18} - i 81_{-8}^{+19}$
Ref. [17], Fit II	$1421_{-2}^{+3} - i 19_{-5}^{+8}$	$1388_{-9}^{+9} - i 114_{-25}^{+24}$
Ref. [18], solution #2	$1434_{-2}^{+2} - i 10_{-1}^{+2}$	$1330_{-5}^{+4} - i 56_{-11}^{+17}$
Ref. [18], solution #4	$1429_{-7}^{+8} - i 12_{-3}^{+2}$	$1325_{-15}^{+15} - i 90_{-18}^{+12}$

Citation: R.L. Workman *et al.* (Particle Data Group), to be published (2022)

$\Lambda(1405) 1/2^-$

$I(J^P) = 0(\frac{1}{2}^-)$ Status: ****

In the 1998 Note on the $\Lambda(1405)$ in PDG 98, R.H. Dalitz discussed the S-shaped cusp behavior of the intensity at the $N\bar{K}$ threshold observed in THOMAS 73 and HEMINGWAY 85. He commented that

Citation: R.L. Workman *et al.* (Particle Data Group), to be published (2022)

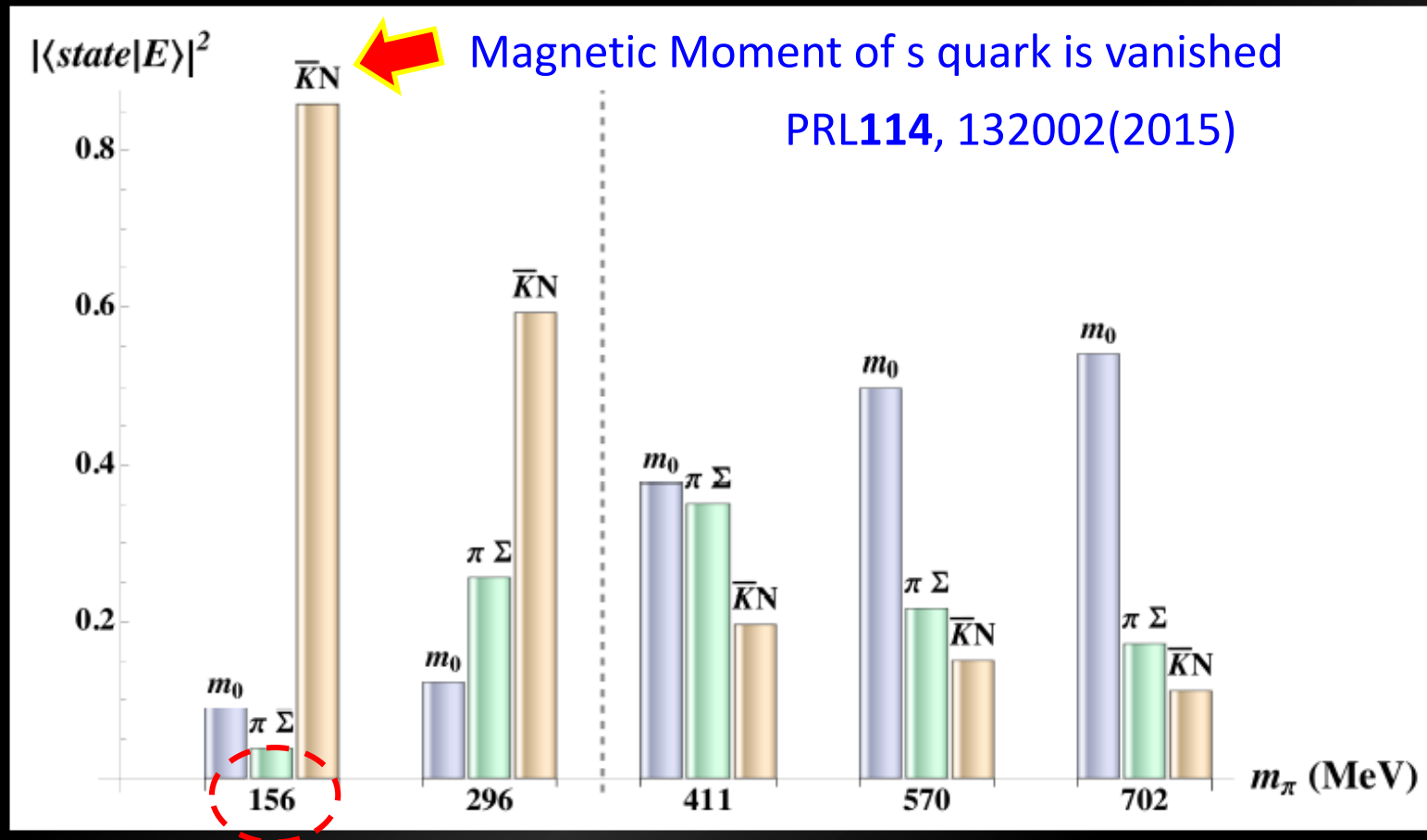
$\Lambda(1380) 1/2^-$

$J^P = \frac{1}{2}^-$ Status: **

OMITTED FROM SUMMARY TABLE

See the related review on "Pole Structure of the $\Lambda(1405)$ Region."

LQCD Evidence that $\Lambda(1405)$ is a $\bar{K}^{\text{bar}}N$ molecule



Dependence of the fitting to the assumed threshold

$$\left. \frac{d\sigma}{dM_{\pi\Sigma}} \right|_{\theta_n=0} \sim \left| T_2^{I'} (\bar{K}N \rightarrow \pi\Sigma) \right|^2 F_{\text{res}}(M_{\pi\Sigma})$$

Threshold	Chi2/ NDF	Scattering Lenth A [fm]	Effective Range R [fm]	Pole [MeV]	$ T_{22}/T_{11} ^2$
K-p	2.03	-1.02(0.11) + 0.92(0.15) <i>i</i>	-0.24(0.38) - 0.50(0.15) <i>i</i>	1416.7 ^{+7.0} _{-7.4} - 28.1 ^{+6.1} _{-7.9} <i>i</i>	1.9 ^{+0.9} _{-0.7}
(K-p+K ⁰ n)/2	1.76	-1.12(0.11) + 0.84(0.12) <i>i</i>	-0.18(0.31) - 0.41(0.13) <i>i</i>	1417.7 ^{+6.0} _{-7.4} - 26.1 ^{+6.0} _{-7.9} <i>i</i>	2.2 ^{+1.0} _{-0.6}
K ⁰ n	1.60	-1.19(0.13) + 0.77(0.14) <i>i</i>	-0.10(0.35) - 0.32(0.12) <i>i</i>	1418.8 ^{+6.4} _{-7.9} - 24.4 ^{+6.6} _{-8.5} <i>i</i>	2.5 ^{+1.1} _{-0.9}
w/ Syst. Err.		-1.12 ± 0.11 ^{+0.10} _{-0.07} + 0.84 ± 0.12 ^{+0.08} _{-0.07} <i>i</i>	-0.18 ± 0.31 ^{+0.08} _{-0.06} - 0.41 ± 0.13 ^{+0.09} _{-0.09} <i>i</i>	1417.7 ^{+6.0+1.1} _{-7.4-1.0} - 26.1 ^{+6.0+1.7} _{-7.9-2.0} <i>i</i>	2.2 ^{+1.0+0.3} _{-0.6-0.3}

次の一手

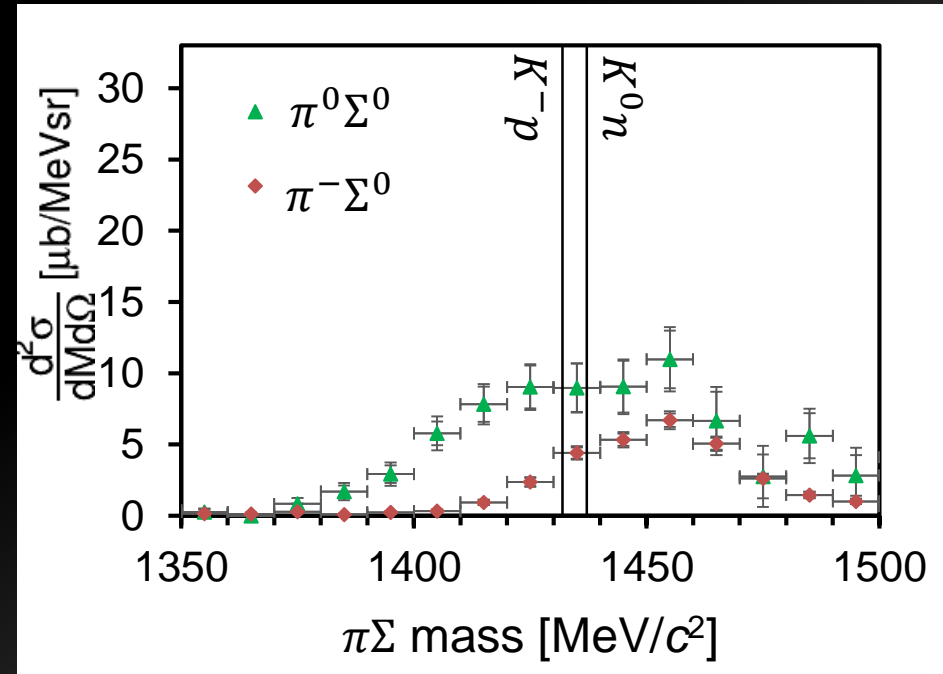
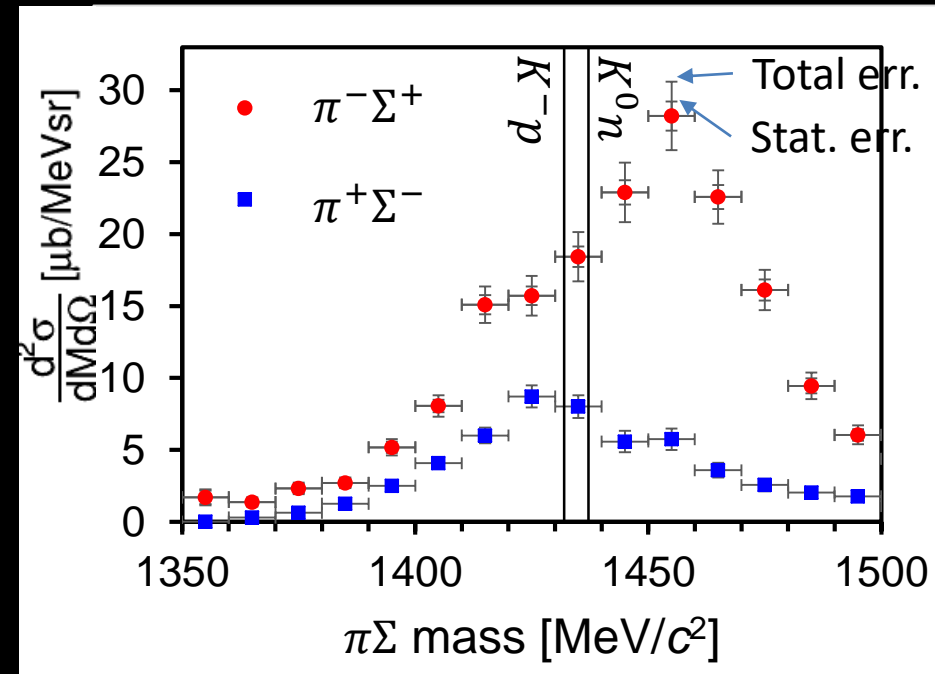
- 広い運動学領域の解析 → 浅野
- ドラミ (より拡大した運動学領域のデータ)
- 3q state or 5q state? → Quark Counting Rule
 - $\pi^- p \rightarrow K^* \Lambda(1405)$
 $d\sigma/d\Omega \sim s^{-(n-2)}$ at large s and t

$$\pi^+\Sigma^-/\pi^-\Sigma^+$$

$$(I' = 0, 1)$$

$$\pi^0\Sigma^0 (I' = 0)$$

$$\pi^-\Sigma^0 (I' = 1)$$



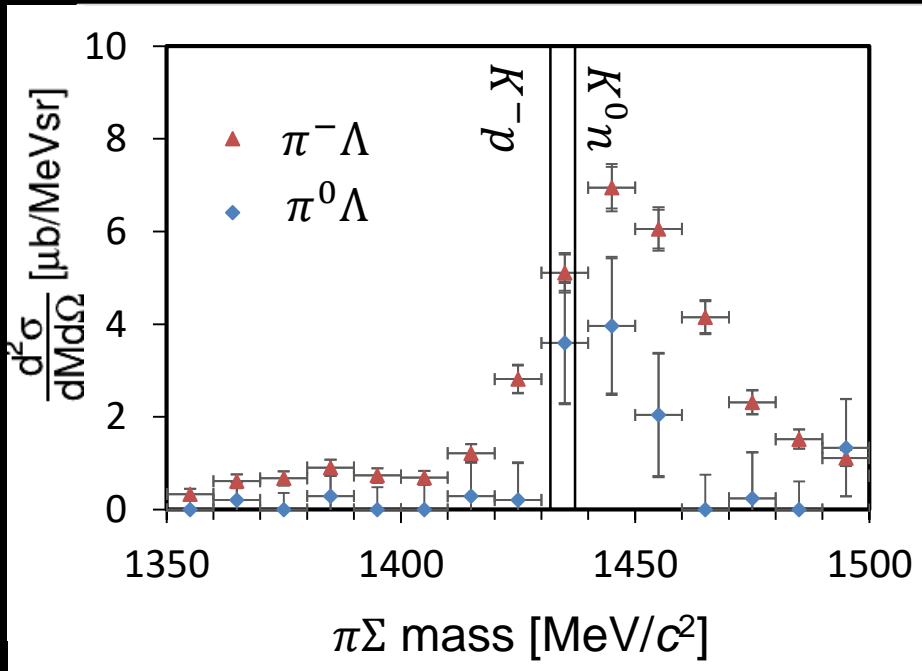
$$\frac{d\sigma}{d\Omega} (\pi^-\Sigma^+/\pi^+\Sigma^-)$$

$$\propto \left| \frac{3T_1^{I=0} - T_1^{I=1}}{4\sqrt{3}} T_2^{I'=0} \pm \frac{T_1^{I=0} + T_1^{I=1}}{4\sqrt{2}} T_2^{I'=1} \right|^2$$

$$\frac{d\sigma}{d\Omega} (\pi^0\Sigma^0) \propto \left| -\frac{3T_1^{I=0} - T_1^{I=1}}{4\sqrt{3}} T_2^{I'=0} \right|^2$$

$$\frac{d\sigma}{d\Omega} (\pi^-\Sigma^0) \propto \left| -\frac{T_1^{I=0} + T_1^{I=1}}{4} T_2^{I'=1} \right|^2$$

$\pi^- \Lambda$ vs $\pi^0 \Lambda$ ($I' = 1$)



$$\frac{d\sigma}{d\Omega}(\pi^- \Lambda) \propto \left| \frac{T_1^{I=0} + T_1^{I=1}}{4} T_2^{I'=1} \right|^2$$

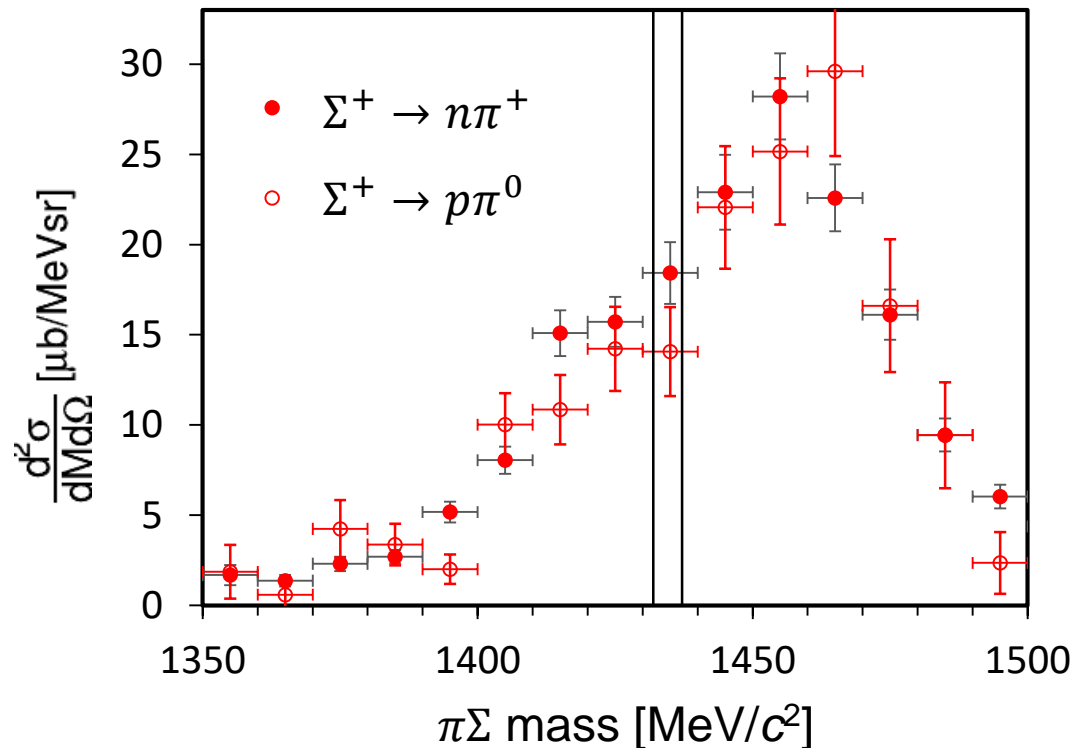
$$\approx 2 \times$$

$$\frac{d\sigma}{d\Omega}(\pi^0 \Lambda) \propto \left| -\frac{T_1^{I=0} + T_1^{I=1}}{4} T_2^{I'=1} \right|^2$$

Isospin relation seems to be satisfied.

Cross Section of $d(K^-, n) \Sigma^+ \pi^-$

$\Sigma^+ \rightarrow p\pi^0$ (by Kawasaki) vs $\Sigma^+ \rightarrow n\pi^+$ (by Inoue)



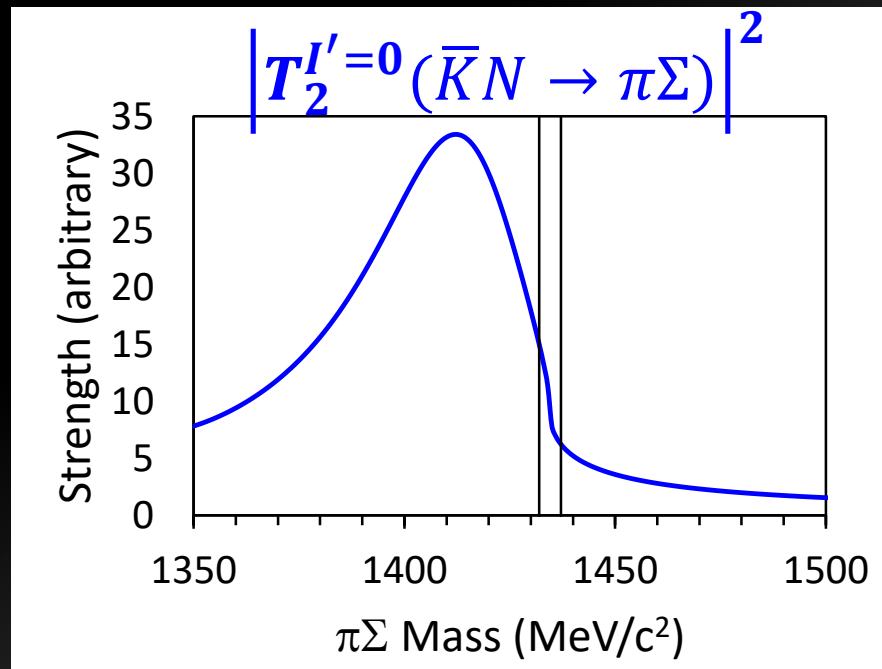
- The CS of $d(K^-, n) \Sigma^+ \pi^-$ measured in the $\Sigma^+ \rightarrow p\pi^0$ decay seems to be identical to that in $\Sigma^+ \rightarrow n\pi^+$.

deduced $\bar{K}N$ scattering amplitude

$$\left. \frac{d\sigma}{dM_{\pi\Sigma}} \right|_{\theta_n=0} \sim \left| T_2^{I'}(\bar{K}N \rightarrow \pi\Sigma) \right|^2 F_{\text{res}}(M_{\pi\Sigma})$$

Scattering Length $A^{I'=0} = (-1.12 \pm 0.11_{-0.07}^{+0.10}) + i(0.84 \pm 0.12_{-0.07}^{+0.08})$ fm

Effective Range $R^{I'=0} = (-0.18 \pm 0.31_{-0.06}^{+0.08}) + i(0.41 \pm 0.13_{-0.09}^{+0.09})$ fm



$\Lambda(1405) : 1405.1^{+1.3}_{-0.9} \text{ MeV}$ (PDG in 2022)

$J^P = \frac{1}{2}^-$, $I = 0$, $M_{\Lambda(1405)} < M_{\bar{K}N}$, lightest in neg. parity baryons



$\Sigma^*(1385), 3/2^+$

$\Lambda(1520), 3/2^-$

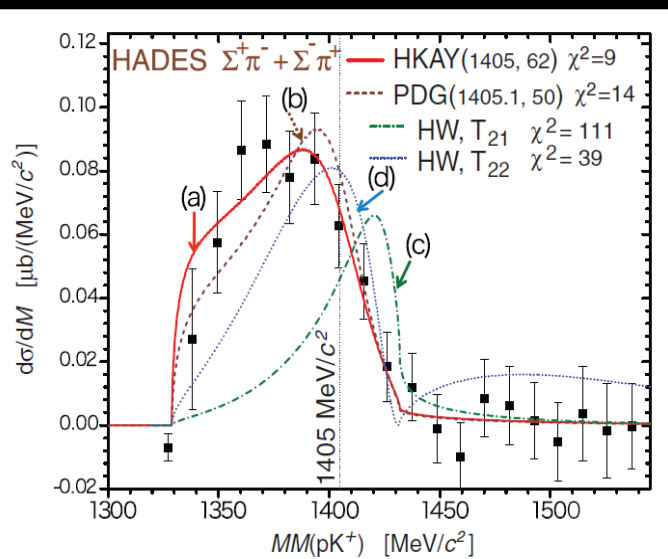
$\Lambda(1405), 1/2^-$

$\bar{K}N(1432)$

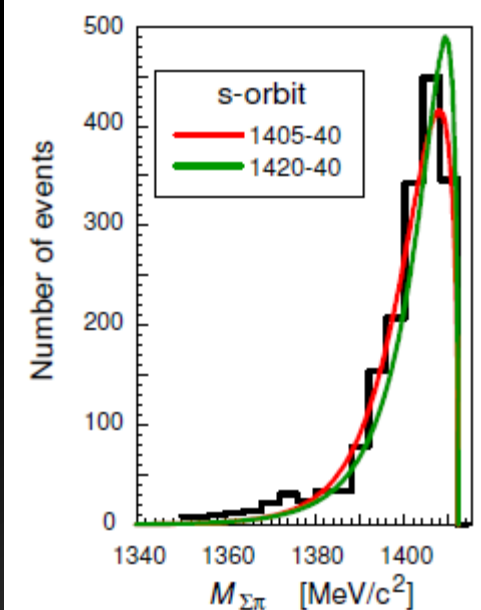
-27 MeV

$\Sigma(1192), 1/2^+$

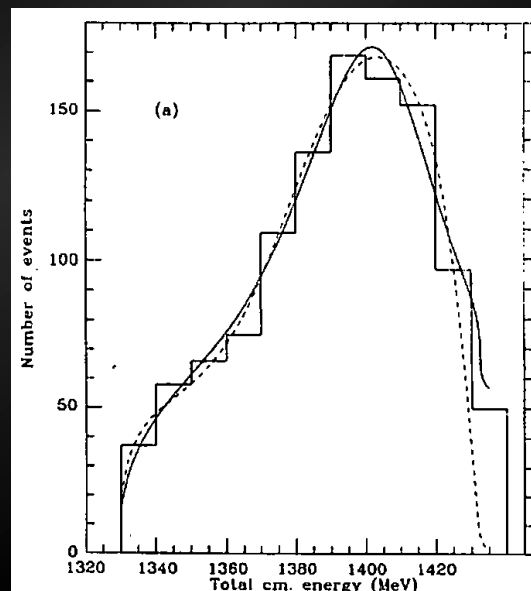
$\Lambda(1116), 1/2^+$



M. Hassanvand et al: $\pi\Sigma$ IM Spec. of $pp \rightarrow K^+\pi\Sigma$



J. Esmaili et al: $\pi\Sigma$ IM Spec. of Stopped K^- on ^4He



R.H. Dalitz et al: $\pi\Sigma$ IM Spec. in $K-p \rightarrow \pi\pi\Sigma$ w/ M-matrix

$\Lambda(1405)$: Double pole?

$J^P = \frac{1}{2}^-$, $I = 0$, $M_{\Lambda(1405)} < M_{\bar{K}N}$, lightest in neg. parity baryons

