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Formation of a Ξ-Hypernucleus and Transitions to Double-Λ States

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A scenario is given for the formation of Ξ^- states and the transitions to states with double-Λ in anticipation of observations, especially in the KEK-E224 experiment. First, the production cross sections of Ξ^- hypernuclear states by (K^-, K^+) reactions are calculated within the framework of the distorted-wave impulse approximation. Next, the transition rates from Ξ^- hypernuclear states to possible double-Λ states are obtained, which are closely related to single- and double-Λ emissions after the $\Xi^- p \to \Lambda\Lambda$ conversion in nuclei.

§ 1. Introduction

Recently, a new experiment concerning the (K^-, K^+) reaction was carried out using a scintillating fiber target; the data analysis is now in progress (KEK-E224).[1] We remark here that the interactions and dynamics between the produced Ξ^- particle and the target nucleus (^{12}C) can be studied from this experiment, noting the possibility of observing Ξ^- hypernuclear states as well as their transitions to double-Λ states. The possibility of forming Ξ hypernuclei via the (K^-, K^+) reaction has been investigated,[2] in which the forward cross sections for the formation of discrete Ξ hypernuclear states were simply estimated to be at the level of only 1 μb/sr. It is expected, however, that in the E224 experiment the production counts in the bound-Ξ^- region have been stored in sufficient number to observe possible peak structures. It is therefore important to perform more realistic calculations of the production cross sections of Ξ hypernuclear bound states under the kinematics of this experiment.

Once a Ξ^- hypernuclear state is formed, it should decay into some two-Λ states through the $\Xi^- p \to \Lambda\Lambda$ transition in the nucleus. There are three types of final $\Lambda\Lambda$ states: two-Λ bound, one-Λ bound and one-Λ continuum, two-Λ continuum. The analysis of the E224 data makes it possible to distinguish these three final states by observing Λ particles produced at the (K^-, K^+) reaction points. The observation of Λ particles from a Ξ hypernuclear state may be useful for obtaining information concerning the Ξ^- nucleus potential, even if no peak structure can be seen in the K^+ spectrum.

In this paper we discuss the calculation of the (K^-, K^+) cross sections for producing Ξ hypernuclear states within the framework of the distorted-wave impulse approximation (DWIA), and estimate the transition rates from Ξ hypernuclear states to possible $\Lambda\Lambda$-sticking nucleon-hole states. The essential points of this work were reported in Ref. 3).

§ 2. Production of Ξ-hypernuclei by the (K^-, K^+) reaction

It is well known that the conventional DWIA for in-flight (K^-, π^-) and (π^+, K^+) reactions can reproduce the absolute cross sections of typical peaks as well as the overall structure of both Λ-hypernuclear excitation functions.[2),4)~6)] This is mainly due to the fact that the relevant meson momenta are sufficiently large (typically $p>700$ MeV/c) to be suitable for an impulse approximation. Based on this fact, we applied DWIA to the in-flight (K^-, K^+) reaction in order to predict the Ξ-hypernuclear production cross sections for the ${}^AZ(K^-, K^+){}^A_\Xi Z'$ reaction, which are concerned with the E224 experiment at KEK (^{12}C target). The DWIA differential cross section and the strength function $S(E_Y, \theta)$ are expressed in the Kapur-Peierls method[5)] by

$$\frac{d^2\sigma}{d\Omega dE_Y}=\alpha\left[\frac{d\sigma(\theta)}{d\Omega}\right]_{\mathrm{elem}}S(E_Y, \theta) \quad \text{and} \quad S(E_Y, \theta)=-\frac{1}{\pi}\sum_f \mathrm{Im}\left[\frac{N_f(E_Y, \theta)}{E_Y-\epsilon_f(E_Y)}\right]. \tag{2·1}$$

Here α is the kinematical factor connecting the 2-body and A-body frames. E_Y is the hyperon energy and a suffix f represents a hypernuclear final state, which may be expressed by the eigenfunction $\Psi_f(E_Y)\equiv|{}^A_\Xi Z';\ E_Y,\ J_f T_f \tau_f \alpha\rangle$ and the eigenenergy $\epsilon_f(E_Y)$.

With Ψ_f and the nuclear target wave function Φ_0, the effective nucleon number $N_f(E_Y;\ \theta)$ in Eq.(2·1) is defined as

$$N_f(E_Y, \theta)=\langle\Phi_0|\hat{O}^\dagger(\theta)|\Psi_f(E_Y)\rangle\langle\tilde{\Psi}_f(E_Y)|\hat{O}(\theta)|\Phi_0\rangle\,. \tag{2·2}$$

Here the transition operator $\hat{O}(\theta)$ is given by the following relevant meson waves:

$$\hat{O}^{(K^-,K^+)}=\int d^3\boldsymbol{r}\,\chi^*_{K^+}(\boldsymbol{k}_f, a\boldsymbol{r})\chi_{K^-}(\boldsymbol{k}_i, \boldsymbol{r})\sum_{\nu=1}^{A} V_-^2(\nu)\delta\left(\boldsymbol{r}-\frac{M_c}{M_A}\boldsymbol{r}_\nu\right), \tag{2·3}$$

where $a\equiv M_A/M_H$ and $M_c=M_H-m_Y$, and square of the V-spin lowering operator V_- converts a proton into Ξ^-. The recoil effect is taken into account in view of the large momentum transfer involved. By definition, the strength function $S(E_Y;\ \theta)$ covers not only the hypernuclear bound states, but also the continuum final states, which consists of the hyperon resonances and the quasi-free processes (QF). Specifically for the bound states ($E_Y<0$) which are mainly concerned here, Eq. (2·1) tends to the ordinary expression with the effective proton number Z_{eff}:

$$\frac{d\sigma(\theta)}{d\Omega_L}=\alpha\left[\frac{d\sigma(\theta)}{d\Omega_L}\right]_{K^-p\to\Xi^-K^+}Z_{\mathrm{eff}}(i\to f;\ \theta)\,. \tag{2·4}$$

Corresponding to the E224 experiment, we confine ourselves to the ^{12}C case. As for the nucleon radial wave functions, we first use the harmonic oscillator (HO) functions with the standard size parameter ($b_N=1.65$ fm), and then the appropriate Woods-Saxon (WS) potential. Here, the WS potentials for N, Λ and Ξ (used in this paper) are represented as

$$U_B(r) = V_0^B f(r) + V_{LS}^B\left(\frac{\hbar}{m_\pi c}\right)^2 (\boldsymbol{l}\cdot\boldsymbol{s})\frac{df(r)}{rdr} + U_{\rm COUL}(r) \qquad (2\cdot5)$$

with $f(r)=1/\{1+\exp[(r-R)/a]\}$ and $R=r_0(A-1)^{1/3}$ fm ($B=N, \Lambda, \Xi$). For the Ξ potential, we adopt the values $V_0^\Xi=-24$ MeV, $V_{LS}^\Xi=1.0$ MeV, $r_0=1.1$ fm and $a=0.65$ fm (taken mostly from Ref. 2)). We also try to change the depth V_0^Ξ considering the uncertainties of our present knowledge.

We simply start with the meson plane waves (PW) to obtain the reference values of the production cross sections. In order to calculate the meson distorted waves (DW), we adopt the eikonal approximation by using the 2-parameter-Fermi type nuclear density $\rho_N(r)$ and the following meson-nucleon total cross sections based on the experiment:[7)]

$$\sigma_{K^-p}=32.5\text{ mb}, \quad \sigma_{K^-n}=25.5\text{ mb}, \quad \sigma_{K^+p}=19.6\text{ mb} \quad \text{and} \quad \sigma_{K^+n}=20.1\text{ mb},$$

which correspond to an incident momentum of $p_{K^-}=1.60$ GeV$/c$ and a scattering angle of $\theta_{K^+}=0°$. It is notable that an approximation based on the empirical meson-nucleon cross sections works satisfactorily for simulating the Klein-Gordon solutions.[6)]

Figure 1 shows the excitation function calculated for the ^{12}C$(K^-, K^+)^{12}_{\Xi}$Be reaction with $p_{K^-}=1.6$ GeV$/c$ and $\theta_{K^+}=0°$. Table I lists the detailed effective proton numbers for the four cases: combinations of the proton wave function (HO vs WS) with the plane and distorted meson waves (PW vs DW). Note that the Woods-Saxon potential with $V_0^\Xi=-24$ MeV accommodates three Ξ^- bound states in $^{12}_{\Xi}$Be of $0s_{1/2}$, $0p_{3/2}$ and $0p_{1/2}$.

Since the hyperon recoil momentum is considerably large (about 500 MeV$/c$), the kinematical condition should cause a preferential excitation of the J-stretched states, such as $|[0p^{-1}0p^\Xi]J=2^+\rangle$. On the other hand, the radial wave function of the $0p^\Xi$ state extends more outside than in the Λ case, due to the shallower potential for Ξ. Thus, in contrast to the (π^+, K^+) spectrum, the different radial behavior from that of the $0p_{3/2}$ proton reduces the 2^+ strengths to be comparable to the $[0p^{-1}0s^\Xi]J=1^-$ strength. This explains the apparently increased 1^- peak of the ground state in relative comparison with the 2^+ peak at $E_\Xi=-2$ MeV. In Fig. 1 we use the smearing width of each peak, which simulates the $\Xi^- p\to\Lambda\Lambda$ conversion width Γ_Ξ obtained in the following section. It is remarkable that the conversion widths are rather small compared with the Σ case.

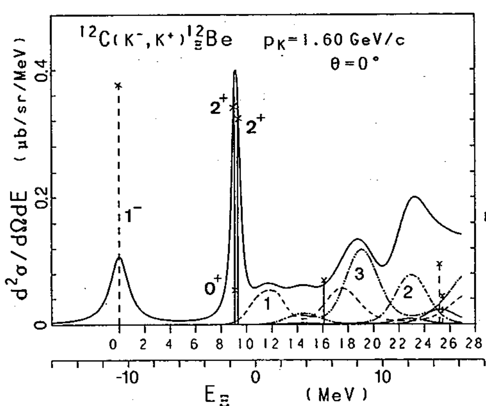

Fig. 1. The excitation function $d^2\sigma/d\Omega dE$ (solid curve) for the ^{12}C$(K^-, K^+)^{12}_{\Xi}$Be reaction with $p_{K^-}=1.6$ GeV$/c$ and $\theta_{K^+}=0°$. The solid (J^+) and dashed (J^-) straight lines show the calculated effective proton numbers drawn relatively to the maximum strength indicated on the right. The conversion width Γ_Ξ and the proton hole width are both taken into account as the smearing width of each state. In the continuum region ($E_\Xi>0$), dashed and dash-dotted curves show partial contributions from quasi-free resonances with $L=1^-$, 2^+ and 3^-, respectively.

Table I. Effective proton number $Z_{\rm eff}$ calculated for the ^{12}C(K^-, K^+)$^{12}_{\Xi}$Be reaction with $p_{K^-}=1.6$ GeV/c and $\theta^L_{K^+}=0°$. The harmonic oscillator (HO; $b_N=1.65$ fm) and Woods-Saxon (WS) potentials were employed to generate the bound-state single particle wave functions for the $[j_N^{-1}j_\Xi]_J$ configuration. DW and PW denote the results of the eikonal K-meson distortion and those of the plane wave approximation, respectively. The last column lists the cross sections (in μb/sr) which were estimated from $Z^{\rm DW}_{\rm eff}$ by multiplying the experimental elementary cross section $\alpha(d\sigma/d\Omega)_{K^-p\to\Xi^-K^+}=22-35$ μb/sr [Ref. 2)].

proton	Ξ^-	$J^\pi(\epsilon_\Xi-\epsilon_p)$	ϕ_N(HO)$\times\phi_\Xi$(WS) $Z^{\rm DW}_{\rm eff}[Z^{\rm PW}_{\rm eff}]$	ϕ_N(WS)$\times\phi_\Xi$(WS) $Z^{\rm DW}_{\rm eff}[Z^{\rm PW}_{\rm eff}]$	$(d\sigma/d\Omega)^{\rm DW}$
		(MeV)	$\times10^{-3}$	$\times10^{-3}$	(μb/sr)
$0p_{3/2}$	→ $0s_{1/2}$	1^-(0.19)	7.50[20.99]	9.60[27.52]	0.211−0.336
	→ $0p_{3/2}$	0^+(9.18)	1.45[5.21]	1.41[5.36]	0.031−0.049
		2^+(9.18)	7.54[19.76]	8.72[23.44]	0.192−0.305
	→ $0p_{1/2}$	2^+(9.46)	7.06[18.48]	8.19[22.01]	0.180−0.287
$0s_{1/2}$	→ $0s_{1/2}$	0^+(16.30)	1.22[3.35]	1.78[5.04]	0.039−0.062
	→ $0p_{3/2}$	1^-(25.29)	2.20[6.08]	2.96[8.43]	0.065−0.104
	→ $0p_{1/2}$	1^-(25.57)	1.02[2.82]	1.38[3.93]	0.030−0.048
Sum over the bound states:			27.99[76.69]	34.04[95.73]	0.748−1.192

$Z^{\rm Total}_{\rm eff}=2.99$ for DW[6.00 for PW].

When one compares the DW and PW results, it can be seen that the meson wave distortion gives rise to a 1/2.8 reduction for the effective proton number estimates. However the amount of such a reduction depends considerably on the size of the nucleus and, in fact, the reduction factors are 1/5−1/6 for the ^{28}Si and ^{40}Ca cases.[8] One sees in Table I that the use of the proton WS wave function causes a sizable increase in the calculated effective numbers.

Based on the eikonal approximation, the total effective proton number is calculated to be $Z^{\rm Total}_{\rm eff}=2.99$ (DW), which should be compared with $Z=6$. In this respect, it is interesting to estimate the strength summed over the bound states; we obtained $\sum_{\rm bound}Z^{\rm DW}_{\rm eff}=(2.8-3.4)\times10^{-2}$. This means that the probability of producing Ξ^- directly in the bound states is theoretically about 1% of the total Ξ^- produced in the in-flight (K^-, K^+) reaction on the ^{12}C target.

In the last column of Table I we add the calculated cross sections estimated with the experimental elementary cross sections which scatter into a fairly wide range,[2] so that we use $\alpha(d\sigma/d\Omega)_{K^-p\to\Xi^-K^+}=22-35$ μb/sr to show the range of the predicted values. Here we note that the recent data at KEK by T. Iijima et al.[9] leads to $\alpha(d\sigma/d\Omega)_{K^-p\to\Xi^-K^+}\simeq0.73\times35\simeq26$ μb/sr. The cross section for the pronounced peak of $[0p^{-1}0p^\Xi]$ obtained at 9 MeV excitation ($0_1^++2_1^++2_2^+$) is predicted to have about 0.5 μb/sr at the forward angle. The value is about 1/40 of the corresponding strength producing Λ in the (π^+, K^+) reaction. In addition to the substantial difference in the elementary cross sections (26 μb/sr vs 500 μb/sr), the reduction in $Z_{\rm eff}$ comes from the larger momentum mismatch and the different radial behavior of the Ξ^- wave function. If we use a deeper potential with $V_0^\Xi=-30$ MeV, the cross section leading to each bound state increases by about 50%: e.g., $d\sigma/d\Omega(1^-_{gs})=0.44$ μb/sr (cf. Table I). On the other hand, if we use $V_0^\Xi=-12$ MeV, we obtain only one bound $0s^\Xi_{1/2}$ state with $d\sigma/d\Omega(1^-_{\rm gs})=0.13$ μb/sr, one half of the ground-state cross section listed in Table I. Thus, the theoretical cross sections depend appreciably on the potential strength V_0^{Ξ}.

The calculation was similarly performed for the ^{16}O$(K^-, K^+)^{16}_{\Xi}$C reaction, and four pronounced peaks were obtained which are well separately from each other. The main feature discussed concerning the effective proton numbers for ^{12}C(K^-, K^+) $^{12}_{\Xi}$Be is also applicable to the latter case. We add, however, that the distortion effect increases so as to result in a smaller ratio $Z_{\rm eff}^{\rm DW}/Z_{\rm eff}^{\rm PW}=1/3.5$ for $^{16}_{\Xi}$C (1/2.8 for $^{12}_{\Xi}$Be), and that the difference between the WS and HO proton wave functions becomes larger. The summed strength over the bound states was also evaluated to be $\sum_{\rm bound} Z_{\rm eff}^{\rm DW}=(2.8-4.0)\times 10^{-2}$, while the total effective proton number is $Z_{\rm eff}^{\rm Total}=3.52$ (DW). Thus, the probability of populating the Ξ-hypernuclear bound states is again predicted to be about 1% of the Ξ^- produced in the in-flight (K^-, K^+) reaction.

§ 3. The $\Xi^- p \to \Lambda\Lambda$ conversion in a nucleus

Here we are concerned with the Ξ^- particle in a nuclear bound state $(n_\Xi l_\Xi)$ which reacts with one of the nucleons to produce two Λ particles. This process is induced by the ΞN-$\Lambda\Lambda$ strong interaction $v_{\Xi N,\Lambda\Lambda}$. We now discuss the calculation of the partial conversion widths (Γ_{bb}, Γ_{bc} and Γ_{cc}, where bb, bc and cc mean that two Λ's in bound states, one Λ in a bound state and the other Λ in a continuum state, and two Λ's in continuum states, respectively). Their sum leads to the total conversion width: $\Gamma_\Xi=\Gamma_{bb}+\Gamma_{bc}+\Gamma_{cc}$. The corresponding probabilities are thus given by $P_{bb}=\Gamma_{bb}/\Gamma_\Xi$, $P_{bc}=\Gamma_{bc}/\Gamma_\Xi$ and $P_{cc}=\Gamma_{cc}/\Gamma_\Xi$, respectively. In the second-order perturbation we have

$$\begin{aligned}\Gamma_{bb}(n_\Xi l_\Xi)=&\sum_{n_N l_N}\sum_{n_{\Lambda_1} l_{\Lambda_1}}\sum_{n_{\Lambda_2} l_{\Lambda_2}}\sum_{LST}\frac{[L][S][T]}{[l_\Xi][s_\Xi][t_\Xi]}\mathcal{P}(n_N l_N)\\ &\times\langle n_\Xi l_\Xi n_N l_N|v_{\Xi N,\Lambda\Lambda}|n_{\Lambda_1} l_{\Lambda_1} n_{\Lambda_2} l_{\Lambda_2}\rangle_{LST}^2\\ &\times\frac{\Gamma_{n_N l_N}^{(h)}}{(\epsilon_\Xi+\epsilon_{n_N l_N}-\epsilon_{n_{\Lambda_1} l_{\Lambda_1}}-\epsilon_{n_{\Lambda_2} l_{\Lambda_2}}+\Delta)^2+(\Gamma_{n_N l_N}^{(h)})^2/4}\end{aligned}\qquad(3\cdot1)$$

and

$$\begin{aligned}\Gamma_{bc}(n_\Xi l_\Xi)=&\sum_{n_N l_N}\sum_{n_{\Lambda_1} l_{\Lambda_1}}\sum_{l_{\Lambda_2}}\int_0^\infty dk_{\Lambda_2}\sum_{LST}\frac{[L][S][T]}{[l_\Xi][s_\Xi][t_\Xi]}\mathcal{P}(n_N l_N)\\ &\times\langle n_\Xi l_\Xi n_N l_N|v_{\Xi N,\Lambda\Lambda}|n_{\Lambda_1} l_{\Lambda_1} k_{\Lambda_2} l_{\Lambda_2}\rangle_{LST}^2\\ &\times\frac{\Gamma_{n_N l_N}^{(h)}}{(\epsilon_\Xi+\epsilon_{n_N l_N}-\epsilon_{n_{\Lambda_1} l_{\Lambda_1}}-\hbar^2 k_{\Lambda_2}^2/2m_\Lambda+\Delta)^2+(\Gamma_{n_N l_N}^{(h)})^2/4}\,.\end{aligned}\qquad(3\cdot2)$$

The expression for Γ_{cc} can be written similarly. Here, $|n_\Xi l_\Xi\rangle$, $|n_N l_N\rangle$, $|n_\Lambda l_\Lambda\rangle$ and $|k_\Lambda l_\Lambda\rangle$ denote a Ξ^- hypernuclear state, a proton-hole state, a Λ-bound state and a Λ-continuum state, respectively, and ϵ_Ξ, $\epsilon_{n_N l_N}$ and $\epsilon_{n_\Lambda l_\Lambda}$ are the corresponding single-particle energies. The continuum wave functions are normalized to $\sqrt{2/\pi}\sin(kr-\pi l/2+\delta_l)$ asymptotically. $\mathcal{P}(n_N l_N)$ is a nucleon occupation probability, by which the unclosed nature of the core nucleus is taken into account. The two-Λ states in these expressions are anti-symmetrized and normalized. The Ξ^-p-$\Lambda\Lambda$ mass difference is denoted by Δ. The Breit-Wigner shape is taken for the distribution of a hole-excited state, where $\Gamma^{(h)}_{n_N l_N}$ denotes the width of a hole state. For $v_{\Xi N,\Lambda\Lambda}$ we adopt a simple delta interaction $t_0\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)$ with $t_0=280$ MeV fm^3, which reproduces effectively the ΞN-$\Lambda\Lambda$ coupling strength of the Nijmegen model-D interaction.[10] The calculated values of P_{bb}, P_{bc} and P_{cc} are quite insensitive to t_0, since they are given by the ratios of the above conversion widths.

Now calculations are performed in the cases of ^{12}C and ^{16}O nuclear targets. The single-particle wave functions of N, Λ and Ξ are obtained by the corresponding WS potentials, respectively. The WS parameters for N are taken as $V_0^N=-50.0$ MeV, $V_{LS}^N=15$ MeV ($r_0=1.25$ fm, $a=0.53$ fm), which reproduces adequately the ϵ_{0p} energies. The ϵ_{0s} energies, the calculated values of which are too shallow, are taken artificially as -35 MeV and -40 MeV for ^{12}C and ^{16}O, respectively, as indicated by the proton knock-out reactions. The WS parameters for Λ are taken as $V_0^\Lambda=-32.0$ MeV (^{12}C) and -30.0 MeV (^{16}O), $V_{LS}^\Lambda=0$ ($r_0=1.1$ fm, $a=0.60$ fm). For the Ξ potential we employed the two cases of $V_0^\Xi=-24$ and -12 MeV. Attractive Coulomb forces are also included between Ξ^- and nuclear cores.

In order to estimate the proton-hole width when the Ξ particle reacts on the $0p$-proton in ^{11}B, we expand the ^{11}B state in terms of the ^{10}Be eigenstates as

$$\sum C_{J_ij}(E_i)[\Phi(^{10}\mathrm{Be};J_i,E_i)\otimes\phi_j(p)]\,. \tag{3·3}$$

Then an approximate way of estimating $\Gamma^{(h)}_{n_N l_N}$ is to take the average energy width of the distribution of $|C_{J_ij}(E_i)|^2$. By using the Cohen-Kurath wave function for the ground $3/2^-$ state of ^{11}B, we got $\Gamma^{(h)}_{0p}\simeq 7$ MeV. On the other hand, the width of $0s$ proton-hole state was taken to be 10 MeV, which is indicated experimentally in the proton knock-out reaction. We tried to use other values around these widths and found that the result was not strongly dependent on the choice of $\Gamma^{(h)}$. The above values of $\Gamma^{(h)}$ are used also for ^{15}N. In the calculation of Γ_{cc}, the Breit-Wigner distribution was replaced by the delta function for simplicity.

In Table II, the calculated probabilities P_{bb}, P_{bc} and P_{cc} are given for the $^{12}_{\Xi}$Be and $^{16}_{\Xi}$C in the cases $V_0^\Xi=-24$ and -12 MeV. The values of E_Ξ are the calculated binding energies of Ξ^-, for which we have well-separated $0s_\Xi$ and $0p_\Xi$ states with $V_0^\Xi=-24$ MeV but only a $0s_\Xi$ state with $V_0^\Xi=-12$ MeV. The ones of Γ_Ξ are the calculated conversion widths. One sees that notable values of P_{bb} are obtained and the relative probabilities are sensitive to V_0^Ξ. Therefore, the observation of these quantities, which are possible in the KEK-E224 experiment, can serve as a useful indicator to know Ξ^- hypernuclear bound states.

Table III shows the partial widths $\Gamma_{ij}(n_\Xi l_\Xi\to l_N^{-1}l_\Lambda^i l_\Lambda^j)$ for two-Λ one nucleon-hole final states specified by $l_N^{-1}l_\Lambda^i l_\Lambda^j$ $(i,j=b,c)$ in the case $^{12}_{\Xi}$Be ($V_0^\Xi=-24$ MeV), where l_N^{-1} denotes a nucleon hole state and l_Λ^i a Λ-bound $(i=b)$ or Λ-continuum $(i=c)$ state. We note that the main part of double-Λ sticking comes from the conversion of Ξ^- and $0s$-proton. For instance, $\Gamma_{bb}(0s_\Xi\to p_N^{-1}s_\Lambda^b p_\Lambda^b)$ (0.025 MeV) is far smaller than $\Gamma_{bb}(0s_\Xi\to s_N^{-1}s_\Lambda^b s_\Lambda^b)$ (0.551 MeV). The reason can be understood from the big difference of the matrix elements $\langle v_{\Xi N,\Lambda\Lambda}\rangle$ involved in the former and the latter (1.74 MeV·fm^3 vs 3.45 MeV·fm^3) and also from the difference of the Breit-Wigner part of Eq. (3·1): 0.022 MeV^{-1} vs 0.185 MeV^{-1}. A problem is how to consider the $s_N^{-1}p_\Lambda^b p_\Lambda^b$

Table II. Transition probabilities from ${}^{12}_{\Xi}\mathrm{Be}$ and ${}^{16}_{\Xi}\mathrm{C}$ to final $\Lambda\Lambda$ states. P_{bb}, P_{bc} and P_{cc} denote the probabilities to two Λ's in bound states, one Λ in a bound state and the other Λ in a continuum state, and two Λ's in continuum states, respectively. Two options are employed for the depth of the Ξ-nucleus WS potential. The nucleon WS depth is fixed at $V_0^N=-50$ MeV, giving rise to $\epsilon_{0p}^p=-11.6$ (-14.6) MeV, but the proton $0s$-hole energy ϵ_{0s}^p is adjusted to -35.0 (-40.0) MeV indicated by the proton knock-out reaction experiment.

	V_0^Ξ (MeV)	$n_\Xi l_\Xi$	E_Ξ (MeV)	Γ_Ξ (MeV)	P_{bb}	P_{bc}	P_{cc}
${}^{12}_{\Xi}\mathrm{Be}$	-24.0	$0s_\Xi$	-10.7	1.20	0.50	0.47	0.03
		$0p_\Xi$	-1.5	0.50	0.22	0.54	0.24
	-12.0	$0s_\Xi$	-4.1	1.08	0.14	0.72	0.14
${}^{16}_{\Xi}\mathrm{C}$	-24.0	$0s_\Xi$	-13.6	2.02	0.55	0.45	0.0
		$0p_\Xi$	-4.1	1.08	0.27	0.64	0.09
	-12.0	$0s_\Xi$	-6.0	1.56	0.19	0.76	0.05

Table III. Partial widths $\Gamma_{ij}(n_\Xi l_\Xi \to l_N^{-1} l_\Lambda^i l_\Lambda^j)$ in MeV for two-Λ one nucleon-hole final states specified by $l_N^{-1} l_\Lambda^i l_\Lambda^j$ $(i, j=b, c)$ in the case ${}^{12}_{\Xi}\mathrm{Be}$ with $V_0^\Xi=-24$ MeV. The indices b and c mean the Λ particle in a bound state and in continuum, respectively.

$n_\Xi l_\Xi$	Γ_{bb}		Γ_{bc}		Γ_{cc}	
$0s_\Xi$	$s_N^{-1}s_\Lambda^b s_\Lambda^b$	0.551	$s_N^{-1}s_\Lambda^b s_\Lambda^c$	0.111	$p_N^{-1}s_\Lambda^c p_\Lambda^c$	0.025
	$s_N^{-1}p_\Lambda^b p_\Lambda^b$	0.017	$s_N^{-1}p_\Lambda^b p_\Lambda^c$	0.030	$p_N^{-1}p_\Lambda^c d_\Lambda^c$	0.016
	$p_N^{-1}s_\Lambda^b p_\Lambda^b$	0.025	$p_N^{-1}s_\Lambda^b p_\Lambda^c$	0.281	$p_N^{-1}d_\Lambda^c f_\Lambda^c$	0.000
			$p_N^{-1}p_\Lambda^b s_\Lambda^c$	0.023		
			$p_N^{-1}p_\Lambda^b d_\Lambda^c$	0.118		
(total)		(0.593)		(0.562)		(0.041)
$0p_\Xi$	$s_N^{-1}s_\Lambda^b p_\Lambda^b$	0.104	$s_N^{-1}s_\Lambda^b p_\Lambda^c$	0.115	$s_N^{-1}s_\Lambda^c p_\Lambda^c$	0.000
	$p_N^{-1}s_\Lambda^b s_\Lambda^b$	0.001	$s_N^{-1}p_\Lambda^b s_\Lambda^c$	0.001	$s_N^{-1}p_\Lambda^c d_\Lambda^c$	0.000
	$p_N^{-1}p_\Lambda^b p_\Lambda^b$	0.007	$s_N^{-1}p_\Lambda^b d_\Lambda^c$	0.008	$p_N^{-1}s_\Lambda^c s_\Lambda^c$	0.007
			$p_N^{-1}s_\Lambda^b s_\Lambda^c$	0.001	$p_N^{-1}s_\Lambda^c d_\Lambda^c$	0.001
			$p_N^{-1}s_\Lambda^b d_\Lambda^c$	0.069	$p_N^{-1}p_\Lambda^c p_\Lambda^c$	0.068
			$p_N^{-1}p_\Lambda^b p_\Lambda^c$	0.057	$p_N^{-1}p_\Lambda^c f_\Lambda^c$	0.004
			$p_N^{-1}p_\Lambda^b f_\Lambda^c$	0.017	$p_N^{-1}d_\Lambda^c d_\Lambda^c$	0.035
					$p_N^{-1}f_\Lambda^c f_\Lambda^c$	0.000
(total)		(0.111)		(0.269)		(0.117)

state, since such a highly excited state may not necessarily result in a final double-Λ fragment. However, the contribution to P_{bb}, is not so large.

In the above calculations, the obtained values of Γ_Ξ seem to be considerably small. One reason is that the statistical weight of the $T=0$ ${}^1S_0$ ΞN-$\Lambda\Lambda$ interaction is small, as discussed regarding the G-matrix calculation.[11] In addition, it should be noted that the ΞN-$\Lambda\Lambda$ interaction deduced from the Nijmegen model-D is fairly weaker than that based on model-F. Of course, there still remains a likelihood of having the stronger ΞN-$\Lambda\Lambda$ coupling and the larger value of Γ_Ξ, which may make it difficult to observe 'narrow' peaks of the Ξ^- hypernuclear bound states in the K^+ spectrum. Even so, some useful information will be obtained by observing single- and double-Λ

emissions from possible Ξ^- bound states. In the present treatment the most uncertain is the potential between Ξ and the nucleus, which remarkably affects our results. In other words, we point out a possibility for obtaining the first reliable information concerning the Ξ states in nuclei by comparing our results with the E224 data.

Zhu et al.[12] investigated the production of double-Λ hypernuclei through the atomic Ξ^- capture. In their model the released energy after $\Xi^- p \to \Lambda\Lambda$ conversion is brought out by emitting a neutron. In the present model, it is reasonable to consider that this kind of process is included *effectively* through $\Gamma^{(h)}$, since the hole width $\Gamma^{(h)}(0p)$ in ^{11}B has been estimated from the proton-pick up distribution over the residual nuclear states of ^{10}Be*, which then decays dominantly to the ^{9}Be$+n$ channel.

§ 4. Outlook

In order to intensively investigate $S=-2$ systems, a new experiment (KEK-E224) concerning the (K^-, K^+) reaction was performed with a scintillating fiber target. In anticipation of observations, especially in the KEK-E224 experiment, we calculated the production cross sections of the Ξ^- hypernuclear states, as well as the transition rates from Ξ^- hypernuclear to possible double-Λ states which are closely related to single- and double-Λ emission probabilities. We point out a possibility for obtaining the first reliable information concerning the Ξ-hypernuclear states and their strong decay to double-Λ states by comparing our results with the E224 data. The two-Λ bound and proton-hole excited state, which are produced with sizable probabilites in our calculation, has to break up into some fragments including double-Λ hypernuclei. An interesting possibility is to identify such a fragment by observing the characteristic π^- decay.[13]

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