

「J-PARCハドロン物理の将来研究  
計画を考える」研究会

**J-PARCと格子QCD**

**– Nuclear Bound Charmonium –**

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# Nuclear-Bound Charmonium

— [ Brodsky, Schmidt, de Teramond, PRL 64 (1990) 1011

— **Different quark flavors:** (up, down vs. charm)

— Attractive QCD van der Waals interaction

— No Pauli blocking = No short-range repulsion

— **The formation of charmonium bound to nuclei ( $A \geq 3$ )**

— A genuine QCD effect at the nuclear level

# Nuclear-Bound Charmonium

— [ Brodsky, Schmidt, de Teramond, PRL 64 (1990) 1011

— **Assumption 1**: interaction between  $c\bar{c}$  and nucleon (N)

— 
$$V_{(c\bar{c})N}(r) = -\gamma \frac{e^{-\alpha r}}{r} \quad (\gamma = 0.4 - 0.6, \alpha = 0.6 \text{ GeV})$$

— parameters determined by the Pomeron model

— **Assumption 2**: interaction between  $c\bar{c}$  and nucleus (A)

— the potential strength is linear in A (Pomeron model)

# Nuclear-Bound Charmonium

Dependence of  $c\bar{c}^{\text{bar}}$  and nucleus interaction

the binding energy of  $\eta_c$ - $^3\text{He}$  [MeV]

$\gamma$	Original work	Folding model <sup>1</sup>	4-body calculation <sup>2</sup>
0.4	-3.0	—	-1.4
0.6	-19.0	- 0.8	-12.6

Folding potential

$$V_{(c\bar{c})A}(r) = \int V_{(c\bar{c})N}(r - r') \rho_A(r') d^3 r'$$

<sup>1</sup> Wasson et al., PRL 67 (91) 2237

<sup>2</sup> Belyaev et al., NPA 780 (06) 100

# Charmonium-Nucleon interaction

— [ Charmonium-Nucleon interaction = “Multi-gluon exchange”

— Color van der Waals interaction given by two-gluon exchange

— Attractive long-range potential behaves like  $1/r^n$  ( $n \geq 6$ )

— the scattering length  $a = 0.24$  fm (Brodsky-Miller<sup>1</sup>)

— QCD sum rules:  $a = 0.1$  fm (Hayashigaki<sup>2</sup>)

— Lattice QCD:  $a = 0.71 \pm 0.48$  fm (Yokokawa-Sasaki-Hatsuda-Hayashigaki<sup>3</sup>)

Note that “ $a > 0$ ” means attractive

<sup>1</sup> Brodsky, Miller, PLB 412 (97) 125

<sup>2</sup> Hayashigaki, PTP 101 (99) 923

<sup>3</sup> Yokokawa et al., PRD 74 (06) 034504

# $c\bar{c}$ -Nucleon potential

$$V_{(c\bar{c})N}(r) = -\gamma \frac{e^{-\alpha r}}{r} \quad (\gamma = 0.4 - 0.6, \alpha = 0.6 \text{ GeV})$$

— [ Born approx.  $a_B = -2M_{\text{red}} \int dr r^2 V_{(c\bar{c})N}(r) = 2M_{\text{red}} \gamma / \alpha^2$

$\gamma$	$a_B$ Born		
0.4	0.16 fm		
0.6	0.47 fm		

$a = 0.24$  fm (color van der Waals)

$a = 0.1$  fm (QCD sum rules)

$a = 0.71 \pm 0.48$  fm (Lattice QCD)

# $c\bar{c}$ -Nucleon potential

$$V_{(c\bar{c})N}(r) = -\gamma \frac{e^{-\alpha r}}{r} \quad (\gamma = 0.4 - 0.6, \alpha = 0.6 \text{ GeV})$$

Bound state formation condition  $D = 2M_{\text{red}}\gamma/\alpha > 1.679798$

$\gamma$	$a_{\text{Born}}$		D
0.4	0.16 fm		0.477
0.6	0.47 fm		1.430

# $c\bar{c}$ -Nucleon potential

$$V_{(c\bar{c})N}(r) = -\gamma \frac{e^{-\alpha r}}{r} \quad (\gamma = 0.4 - 0.6, \alpha = 0.6 \text{ GeV})$$

[  $\gamma=0.6$  is too strong for charmonium-nucleon interaction

$\gamma$	$a_{\text{Born}}$	$a$	D
0.4	0.16 fm	0.21 fm	0.477
0.6	0.47 fm	1.65 fm	1.430

$a = 0.24$  fm (color van der Waals)

$a = 0.1$  fm (QCD sum rules)

$a = 0.71 \pm 0.48$  fm (Lattice QCD)



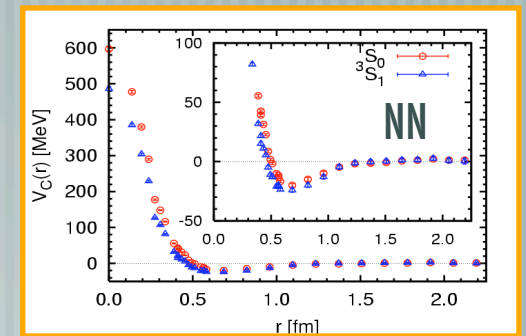
# Toward more realistic prediction

— [ Measure  $cc^{\text{bar}}$  - Nucleon potentials in lattice QCD

— momentum-dep. of phase shift (interaction range  $r$ )

— **Wave Function Approach** (CP-PACS)

c.f. NN potential (Ishii-Aoki-Hatsuda)



— [ Exact few-body calculations with the realistic  $cc^{\text{bar}}$  - N potential

— **Gaussian Expansion Method** (Hiyama-Kino-Kamimura)

# Backup Files

# Lattice results (quench approx.)

— [ Yokokawa, Sasaki, Hatsuda, Hayashigaki, PRD 74 (06) 034504

— Interaction of charmonia with light hadrons is **always attractive**

— the  $cc^{\text{bar}}$  - pion scattering length is very small ( $a = 0.012 \pm 0.004$  fm) as consistent with the soft-pion theorem.

— the  $cc^{\text{bar}}$  - Nucleon (rho) scattering length is an order of magnitude larger than that in the  $cc^{\text{bar}}$  - pion channel

$$0 < a^{(c\bar{c})\pi} \ll a^{(c\bar{c})\rho} < a^{(c\bar{c})N}$$

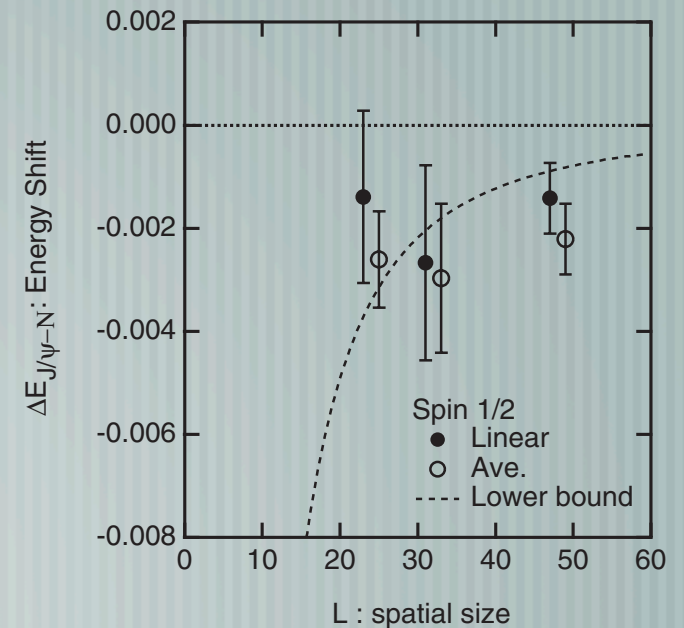
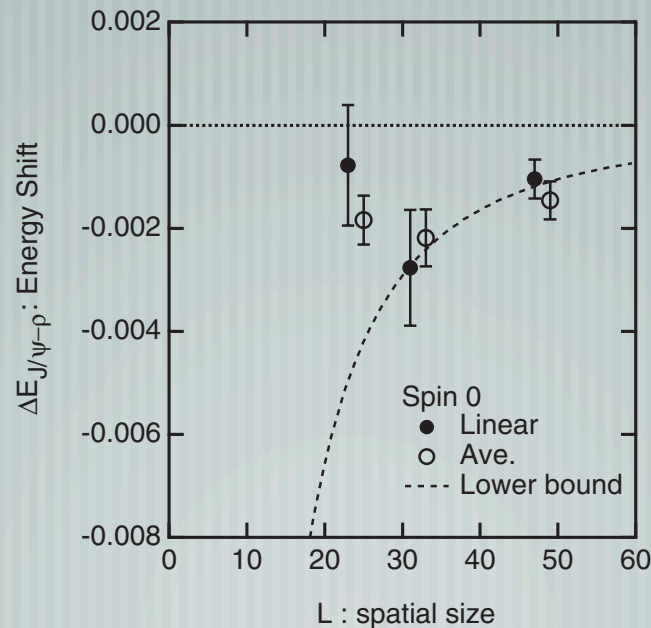
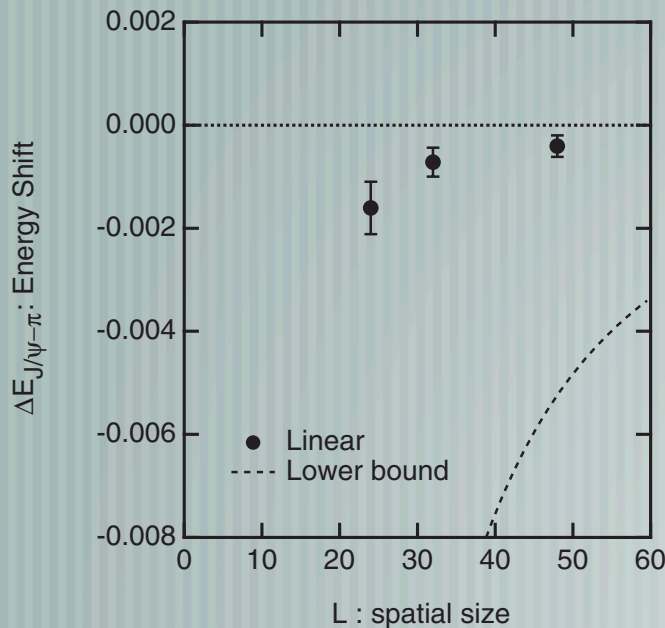
— No appreciable spin-dependence:  $cc^{\text{bar}} = \eta_c$  or  $J/\psi$

# Lattice results (quench approx.)

$\beta$	$a$ [fm]	$a^{-1}$ [GeV]	Lattice size ( $L^3 \times T$ )	$\sim La$ [fm]	Statistics
6.2	0.06775	2.913	$24^3 \times 48$	1.6	161
			$32^3 \times 48$	2.2	169
			$48^3 \times 48$	3.2	53

Lüscher finite size formula

$$\Delta E = -\frac{2\pi a_0}{\mu L^3} \left( 1 + c_1 \frac{a_0}{L} + c_2 \left( \frac{a_0}{L} \right)^2 \right) + O(L^6)$$



Channel	Spin	From PSF		From LLE	
		$a_0$ [fm]	$\sigma_{\text{el}}$ [mb]	$a_0$ [fm]	$\sigma_{\text{el}}$ [mb]
$J/\psi-\pi$	1	$0.0119 \pm 0.0039$	$0.018^{+0.013}_{-0.010}$	$0.0119 \pm 0.0025$	$0.018^{+0.008}_{-0.007}$
$J/\psi-\rho$	0	$0.32 \pm 0.12$	$12.9^{+11.0}_{-7.6}$	$0.23 \pm 0.06$	$6.6^{+4.2}_{-3.2}$
	1	$0.25 \pm 0.10$	$7.9^{+7.3}_{-4.9}$	$0.19 \pm 0.06$	$4.6^{+3.3}_{-2.4}$
	2	$0.21 \pm 0.09$	$5.5^{+5.6}_{-3.7}$	$0.17 \pm 0.06$	$3.5^{+2.8}_{-2.0}$
	SAV	$0.23 \pm 0.08$	$6.8^{+6.1}_{-4.2}$	$0.18 \pm 0.05$	$4.1^{+2.9}_{-2.1}$
$J/\psi-N$	1/2	$0.57 \pm 0.42$	$41^{+83}_{-38}$	$0.35 \pm 0.15$	$15^{+15}_{-10}$
	3/2	$0.88 \pm 0.63$	$96^{+188}_{-89}$	$0.43 \pm 0.16$	$23^{+20}_{-14}$
	SAV	$0.71 \pm 0.48$	$64^{+116}_{-57}$	$0.39 \pm 0.14$	$20^{+16}_{-12}$

Channel	Spin	From PSF		From LLE	
		$a_0$ [fm]	$\sigma_{\text{el}}$ [mb]	$a_0$ [fm]	$\sigma_{\text{el}}$ [mb]
$\eta_c-\pi$	0	$0.0113 \pm 0.0035$	$0.016^{+0.011}_{-0.008}$	$0.0112 \pm 0.0024$	$0.016^{+0.008}_{-0.006}$
$\eta_c-\rho$	1	$0.21 \pm 0.11$	$5.3^{+7.5}_{-4.3}$	$0.16 \pm 0.05$	$3.4^{+2.5}_{-1.8}$
$\eta_c-N$	1/2	$0.70 \pm 0.66$	$62^{+172}_{-62}$	$0.39 \pm 0.14$	$19^{+16}_{-11}$

$L^3 \times T$		$a_0^{\eta_c-\pi}$	$a_0^{\eta_c-\rho}$	$a_0^{\eta_c-N}$
$24^3 \times 48$	Physical	0.15(5)	-0.25(48)	1.10(1.29)
	Chiral	-0.0025(15)	-0.25(48)	1.09(1.28)
$32^3 \times 48$	Physical	0.15(5)	2.8(1.6)	8.8(9.2)
	Chiral	-0.0013(9)	2.7(1.6)	8.7(9.0)
$48^3 \times 48$	Physical	0.33(17)	3.4(1.8)	12.4(10.5)
	Chiral	-0.0014(14)	3.4(1.8)	12.2(10.2)

$$a_0^{H-\pi} = -\left(1 + \frac{M_\pi}{M_H}\right)^{-1} \frac{M_\pi}{4\pi f_\pi^2} \vec{I}_\pi \cdot \vec{I}_H + O(M_\pi^2),$$

where  $\vec{I}_\pi$  ( $\vec{I}_H$ ) is the isospin vector of  $\pi$  ( $H$ )

$$\vec{I}_\pi \cdot \vec{I}_H = 0 \quad \text{for isoscalar hadron}$$

# Possible Experiments

— [ Production of  $\eta_c$ -nucleus bound states



— [ \*Missing mass analysis ( $M_X$ ) on  ${}^3\text{He} + X$

— [ \*Invariant mass analysis ( $M_{\gamma\gamma}$ ) on  ${}^3\text{He} + \gamma\gamma$